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Integrating Commodity Markets in the Procurement Policies for Different Supply Chain Structures

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Integrating Commodity Markets in the Procurement Policies for Different Supply Chain Structures

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Dedicated to my grandmother Late Vidhyawati Goel

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This research develops a mathematical model of procurement for commodities which integrates the arbitrage free pricing models for commodities in Finance with the traditional inventory models of Operations Management. In essence, we develop a model that uses market determined information on spot and futures prices to ascertain the optimal procurement strategy. This research is an attempt to understand how firms should adapt their operating policies in presence of fluctuating commodity prices.

In this research we seek to understand how the term structure of futures prices at a commodity market can be used in the formulation of procurement and distribution policies of supply chains under centralized decision making. The difference between spot and futures prices play an important role in the determination

of the actual cost of holding a commodity; the cost of holding a unit of a tradable commodity is a random variable whose values are determined by the stochastic evolution of prices at the commodity market, and it is exogenously imposed on the firm. The benefits derived from storing a unit of the commodity, on the other hand, are endogenous to each firm and depends on its operational characteristics. Our research objective is to understand how the internal operational decisions of the firm should be modified as a function of the spot and futures prices observed in the market, in order to achieve an optimal balance between cost and benefits of holding an inventory. We model prices with a stochastic process that allows no risk-free arbitrage opportunities, and in this setting, we characterize optimal procurement and distribution policies for various supply chain structures. In addition, we explore the value of using two factor price model over one-factor price model on procurement costs.

Our results suggest that there are substantial cost savings in inventory related costs on incorporating spot and futures price information in the procurement model. Furthermore, two-factor model yields higher cost savings than using a single factor model to forecast prices. Distribution of commodities requires the understanding of price dynamics on the commodity markets as well as the issues related to supply chains. This dissertation is an attempt to contribute to the understanding of this area of research.

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Chapter 1

Executive Summary

Commodities have been traded on organized exchanges for couple of centuries. Chicago Board of Trade (CBOT) is the prime market to trade agricultural commodities such as wheat, oat, and corn. Similarly, New York Mercantile Exchange (NYMEX) specializes in trading of crude oil and related commodities. London Metal Exchange (LME) is one of the prominent marketplace to trade metals such as copper, aluminum, zinc etc. In addition, in the recent past we have witnessed the emergence of several other B2B marketplaces for industrial commodities. These include Converge and DRAM-Exchange for semiconductors, E-steel for steel, and Inter-continental paper exchange for paper related products. In addition, several traditional organized exchanges have started trading industrial commodities. NYBOT and CBOT started trading long term futures contracts for ethanol in the year 2005. Also, NYBOT recently introduced futures and option contracts on wood pulp. London Metal Exchange (LME) started trading futures contracts on plastics, and is deliberating to start trading futures contracts on steel.

In general, the spot price of a commodity reflects the existing dynamics of supply and demand in the market, whereas futures prices reflect the anticipated demand and supply equilibrium in the future. Moreover, the information generated by the futures markets can also be useful in negotiating forward contracts between buyers and suppliers of the commodity since the future price dictated by the market contains more information on demand/supply dynamics than the information held by individual buyers and suppliers. The role of futures market in hedging risk has been very well researched in the literature of Economics and Finance. However, in this research, we seek to understand additional benefits of observing futures price curve of a commodity in the development of procurement policies.

In this research, we seek to understand how the term structure of futures prices at a commodity market can be used in the formulation of procurement and distribution policies of supply chains under centralized decision making. The difference between spot and futures prices play an important role in the determination of the actual cost of holding a commodity; the cost of holding a unit of a traded commodity is a random variable whose values are determined by the stochastic evolution of prices at the commodity market, and it is exogenously imposed on the firm. The benefits derived from storing a unit of the commodity, on the other hand, are endogenous to each firm and depend on its operational characteristics. Our research objective is to understand how the internal operational decisions of the firm should be modified as a function of the spot and futures prices observed in the market. We model prices with a stochastic process that allows no risk-free arbitrage opportunities, and in this setting, we characterize optimal procurement and distribution policies for various supply chain structures. We consider a problem of single location, serial distribution, and multiple location distribution network.

The third chapter of the dissertation deals with a single location problem.

Manufacturers often rely on different types of long term contracts with established suppliers to procure goods often involving delivery lead times. Commodity markets as well as online markets provide additional procurement flexibility; manufacturers can procure through their conventional channels or interact directly with the market either through spot or forward transactions. In this chapter, we explore the value of incorporating information about spot and futures market prices in procurement decision making of a commodity user who satisfies a random demand of a finished product. We also model transaction costs associated with procurement from spot and forward markets. Due to shorter response times, transaction costs associated with spot market procurement (including freight) are typically higher. We develop optimal and approximate procurement policies for this problem, and our results suggest that it is possible to significantly reduce inventory related costs by incorporating spot and futures price information in the procurement decision making process. Our results also imply that, apart from risk reduction, the potential savings in transaction costs associated with forward procurement entice manufacturers to procure a relatively higher fraction of goods from forward markets, using spot procurement only to fine tune stocking levels and recover from emergencies.

The fourth chapter of the dissertation extends the analysis from a single location, and models procurement and distribution of commodity in a serial and distributive supply chain. In this study, the commodity is procured at the central location and then shipped downstream to the retailers. Our analysis illustrate how market determined spot and futures price information can be used to develop an efficient procurement and distribution policy. We obtain an optimal solution for serial distribution system, however, for multiple location distribution system we characterize the optimal solution and develop bounds on the cost function that are robust for non-stationary demand and non-identical retailers. Commodity procurement and distribution has evolved as a big business in the energy industry after the de-regulation of various commodities. This chapter is a step in developing the understanding of supply chains of commodities.

Once the procurement policies have been developed, we empirically evaluate the relevance of such policies in the fifth chapter of dissertation. In particular, we explore the effect of additional term structure information of futures prices on the procurement policies from the spot market. In this regards, we compare the policies developed from one-factor stochastic price model with the policies developed using a two-factor model. In addition, we also explore the benefits of frequent calibration of stochastic price process on the procurement cost structure of a firm. We conclude through our analysis that for gasoline the procurement policies which use futures price forecasts based on a two-factor model yields substantial cost reductions in comparison to policies that use futures price forecasts based on the single factor model. This result entails due to the ability of a two-factor model to capture the time varying marginal convenience yield better than the one-factor model. In addition, we do not observe any advantage of calibrating stochastic price model more frequently as the parameters of the price model for gasoline remain relatively stationary over a period of time.

Chapter 2

Literature Review

There are three areas of operations management literature related to this research; inventory management models allowing multiple modes of supply, inventory management models allowing supply to multiple location, and procurement models considering access to commodity markets.

2.1 Multiple Modes of Supply

Models with multiple modes of supply have been extensively researched in the operations management literature. We refer readers to Fukuda (1964), Whittemore and Saunders (1977), Moinzadeh and Lee (1989), Chiang and Gutierrez (1998), Lawson and Porteus (2000), Tagaras and Vlachos (2001), and references therein. The above papers differ in their consideration of expediting options, inventory review (continuous or periodic), number of echelons, and cost structure (variable and set-up costs) associated with different modes of supply; however, the common theme in this line of research is the consideration of modes of supply with smaller lead times in exchange for an additional known cost. The models trade the additional cost of faster supply in the presence of demand uncertainty against the benefits of lower expected costs of shortage and storage over the smaller lead time.

2.2 Multi-echelon Models

The Operations Management literature has paid considerable attention to research on the problem of devising procurement and allocation policies for multi-echelon inventory systems. Clark and Scarf (1960) obtained the optimal solution for a serial multi-echelon model, and their work lead to a very rich body of literature on multi-echelon models. Fukuda (1964) derived an optimal solution for a problem with dual modes of supply when one mode of supply lags the other by one time period. Zhang (1996) obtained a stationary myopic policy for a dual supply mode under infinite horizon framework. Moinzadeh and Lee (1989) model dual modes of supply in a continuous review setting. In addition, Lawson and Porteus (2000) extend the framework of Clark and Scarf (1960) by modeling regular and expedited modes of supply; they show that top-down base stock policies are optimal, and obtain optimal myopic policy for stationary infinite horizon case.

Two echelon systems involving a central procurement facility as an upper echelon and multiple downstream locations have been traditionally difficult to solve for optimal procurement and distribution policies. Clark and Scarf (1960) illustrated that it is intractable to obtain an optimal solution for the problem due to the large state space of the stochastic models. To deal with this problem, researchers have repeatedly resorted to approximations to obtain near optimal solutions. In this regards, a "balancing of inventory" assumption is very common, this assumption is made on the grounds that it is possible to bring all the retailers to a common normalized inventory level before any shipment is made from the warehouse to the retailers. Eppen and Schrage (1981) study a centralized order system where different demand locations place orders from a central warehouse in fixed cycles; they characterize the optimal ordering policy as (m, y), assuming the probability of stock imbalance as negligible, where m is the period in which order is placed, and y is the inventory position. Federgruen and Zipkin (1984) also apply the balancing assumption for a periodic review finite horizon model, in order to propose a solution for the dynamic program that approximates the original problem by collapsing the large state space to a single variable; allowing for a computational solution. Jonsson and Silver (1987) employs the same assumption, but allows for complete redistribution of inventory during the order cycle to mitigate the perils of imbalance in inventory. Jackson (1988) explores the allocation policy to the retailers in a multi-period model, where a pre-specified quantity of stock available at the warehouse is to be allocated amongst retailers following a base stock policy until the warehouse falls short of stock and fails in maintaining the base stock inventory at the retailers. In this "run out" period, allocation policies are developed assuming balancing of inventory for identical retailers. Erkip et al. (1990) extend the model of Eppen and Schrage (1981) to include the possibility of correlation in demand across retailers and time.

Zipkin (1984) addresses the issue of imbalances in inventory and postulates that for a system with equal coefficients of variation of demand, the approximation of myopic allocation yields near optimal solution. Similarly, Federgruen and Zipkin (1982) illustrate through numerical results that policies based on the balancing assumption for systems with non-identical retailers can lead to significant imbalances in inventory for future periods. Non-optimality of balancing policies in non-identical retailers is also shown by Jackson and Muckstadt (1989), and McGavin et al. (1997). The above described models consider that the price of the commodity remains stationary over a finite horizon. However, this assumption is restrictive in comparison to what actually is observed in the supply chains, thus, in our model, we allow for prices to fluctuate from one period to another. Imbalance of inventory can be exacerbated in environments of non-identical retailers where prices are also fluctuating because it will lead to more dispersion in optimal stocking policy across retailers and time. Significantly, in this research, we seek to develop policies that can account for stochastic prices under a framework of one warehouse and multiple non-identical retailers in order to minimize the supply chain costs. Next, we discuss the emerging literature on procurement from commodity markets.

2.3 Procurement from Commodity Markets

Spot markets provide flexibility for commodity users as it allows them to fine tune their stocking and production policies under uncertain demand and supply environment. Spot markets play an important role in clearing of market through adjustments in prices, which eventually leads to more efficient allocation of resources. As an observation, spot markets have proliferated in energy markets after the de-regulation of natural gas, electricity and other energy related commodities. Traditionally, in commodity markets of copper and oil two-part pricing existed; one was a contract price between the buyer and the manufacturer, and the other was the price listed on commodity exchange Hubbard and Weiner (1989). However, over a period of time contract pricing in copper and oil markets along with many other commodity markets have ceased to exist. Hubbard and Weiner (1989) provide support to their hypothesis through econometric analysis that the importance of spot markets in copper and oil commodity markets is due to the speed of adjustment in prices pertaining to imbalances in demand and supply. As a result, any shock in demand

and supply of a commodity gets reflected in the spot price of the commodity. This leads to fluctuating spot prices, which exacerbates the procurement strategy from the spot market. Spot prices fluctuate more in comparison to the futures prices as postulated by Samuelson (1965) and Fama and French (1988). On one hand spot markets provide flexibility in procurement, on the other hand, they pose a challenge in designing procurement policies due to fluctuating prices. Significantly, we develop procurement policies from spot market under presence of volatile cash prices.

Growth in B2B markets has lead to the development of literature in Operations Management on procurement from spot markets. In particular, researchers have explored the optimal mix of long-term and short-term contracts to reduce cost and enhance flexibility in supply chains. In this regards, Seifert and Thonemann (2004) explore the procurement and selling policies in a spot market under fluctuating prices and random demand. Similarly, Fu et al. (2006) design an optimal portfolio of optimal contracts when demand is uncertain and prices are stochastic. Cohen and Agrawal (1999) also evaluate the trade-off between long-term and short term contracts by developing a bi-nominal model of price variation. They conclude that there is no single dominant strategy of procuring contracts. Akella et al. (2002) model the optimal allocation of supply capacity before observing the realization of a random demand. In their model, if the realized demand is greater than the allocated capacity then the residual demand gets satisfied from the spot market at the market price. Yi and Scheller-Wolf (2003) model procurement under two supply modes where one mode offers a contract at known price, and the other mode of procurement is through a spot market at unknown price. Ritchken and Tapiero (1986) employ contingent claim analysis to mitigate price and demand risk in designing inventory management policies. Mendelson and Tunca (2007) propose an endogenous model of interaction between a single supplier and multiple manufacturers where sourcing is done through a fixed price contract and spot procurement, and describe the strategic interaction amongst the players and its influence on the formulation of fixed-price contract and the supply chain efficiency. Mendelson and Tunca (2007) conclude that increase in the number of manufacturers leads to a shift in procurement from fixed-price contracts to spot procurement, this makes the supply chain fully coordinated and informationally efficient. This result is aligned with the findings of Hubbard and Weiner (1989), and can be explained by the following argument. Since due to globalization the number of potential manufacturers in the market place have increased, we observe the extinction of fixed price contracts from oil and copper markets. Golovachkina (2003) design an option contract between a supplier and a manufacturer with limited capacity, and demonstrate that the channel can be almost coordinated if the margins on spot market are high. Etzion and Pinker (2004) models a spot market that consists of two suppliers, one that sells in a spot market and satisfies stochastic demand from other source, and the second supplier only sells in the spot market. Etzion and Pinker (2004) obtain conditions on the size of the spot market under which one type of supplier benefits over the other. Milner and Kouvelis (2002) study the impact of speculative behavior in non-commodity markets on long-term contracts. They determine the spot price through a market clearing mechanism, including inventory parameters and expected future demand, in a multi-firm environment; thus, demonstrating the influence of the inventory positions of several firms on market prices under an infinite horizon model. Furthermore, Kleindorfer and Wu (2003) provide a comprehensive overview of the literature in the area of B2B exchanges, in particular, their paper focuses on the efficient integration of long term and short term contracts through options on capacity. We also refer readers to Haksoz and Seshadri (2005) for more comprehensive review on spot procurement related research.

Price uncertainty has also been studied by several researchers in the context of inventory policy. Gurnani and Tang (1999) obtain an optimal ordering policy for a retailer who orders a seasonal product from a manufacturer in two stages. In this model, a retailer faces the trade-off between a certain price and uncertain demand in a period versus a stochastic price and improved demand information in a second period. Similarly, Kouvelis and Gutierrez (1997) consider a model with price variability due to exchange rate risk in a news-vendor problem with a two stage sale season in two different countries. Golabi (1985) obtain ordering policies when prices are stochastic and follow a known distribution. Similarly, Wang (2001) investigated inventory replenishment policy under the influence of decreasing prices, and stochastic demand in a multi-period model. Wang (2001) prove myopic policy to be optimal and characterize the optimal procurement policies for the lost sales. However, the problem of obtaining an optimal procurement strategy becomes more complex if the prices follow a continuous distribution. In this context, Li and Kouvelis (1999) study supply contracts to satisfy a deterministic demand under procurement price uncertainty, and evaluate optimal procurement policies for time-flexible and time-inflexible contracts. They illustrate that adequately structured risk-sharing contracts provide opportunities for profit through sourcing from volatile price environments. However, as they comment, this problem gets more complex if demand is stochastic and cost is minimized over multiple periods. Significantly, it is the uncertainty of demand and price of storable commodities in a multi-period environment that we model in this research. Berling and Rosling (2005) explore the impact of financial risk on the procurement policy in a EOQ model, and conclude that major financial risk in the inventory control is due to the systematic purchase price risk, and infer that the systematic risk due to demand has a negligible effect. However, Berling and Rosling (2005) assume a constant convenience yield in determining the financial risk associated with the purchase cost. Significantly, in our research we demonstrate that an inventory model which incorporates stochasticity of convenience yield leads to a substantial savings in inventory related costs over the constant convenience yield model.

Spot markets are very prominent in the electricity markets, and this has lead to a growing body of literature related to spot procurement in electricity markets. Spot electricity prices fluctuate considerably due to the non-storable nature of the electricity. Absence of inventory dissuades smoothening of consumption leading to sharp spikes in prices, for example electricity prices in US Midwest rose from \$30/MWh to \$7000/MWh during June 1998, Trebing (2000). Since electricity cannot be stored arbitrage free pricing methods cannot be used to develop pricing models, thus, Bessembinder and Lemmon (2002) obtain forward prices using an equilibrium approach. Routledge et al. (2001) also obtain forward prices through an equilibrium model where they draw a connection between the natural gas and electricity prices as natural gas can act as an inventory for electricity. Dong and Liu (2003) also investigate forward contracts on non-storable commodities in presence of spot markets. They consider the market power of a risk-averse manufacturer and a supplier in the forward market, and conclude that in non-storable commodities the forward price could be non-monotonic in the spot price.

The US economy consumes around 20 million barrels per day of petroleum products that are transported through a complex network of pipelines that, according to association of oil pipe lines, accounts for 17% of the total transportation volume of the economy. However, the literature related to supply chain management of commodities is very scant in spite of its substantial contribution to the economy. Following we review examples of research on supply chain management of commodities. Reiman and Wein (1999), model the distribution of gasoline from warehouse to the retailers. The focus of their paper is to ascertain transportation policies under a queuing model framework using heavy traffic approximations. One of the first papers to analyze the supply chain of a commodity in equilibrium is written by Tayur and Yang (2002), they analyze the supply chain of natural gas connected by two markets at two ends. Equilibrium and uniqueness of prices are shown as a function of inventory level in these markets. In this regards, Goel and Gutierrez (2007a) propose an optimal inventory policy that incorporates the marginal convenience yield, exogenously determined by the market, in the decision making of an individual firm. They show that incorporating time varying marginal convenience yield in the optimal stocking policy yields substantial reduction in inventory related costs. In addition, Goel and Gutierrez (2007b) demonstrate that price information on spot and forward contracts observed in the commodity markets can be used to obtain better procurement and distribution policies in a supply chain. Similarly, Secomandi (2007) characterize the optimal policy for storage of natural gas as a two price state variable dependent base stock levels. Natural gas storage technology requires inventory levels to determine the capacity function on the storage facility, thus the operational aspects of inventory management can influence the trading policy of a natural gas merchant. In a related research, Wang et al. (2007) develop a stochastic-dynamic model to determine an optimal release policy of natural gas from a downstream re-gasification facility into the whole sale market. Distribution of commodities is operationally intensive and require the combined understanding of financial markets and supply chain issues. Unfortunately, there is not much literature on issues dealing with supply chain of commodities. This research is an attempt to enrich this small body of emerging literature.

Chapter 3

Optimal Procurement Policies for a Single Echelon

3.1 Introduction

Procurement and inventory management models in the operations management and supply chain management literature usually assume constant or known procurement prices while modeling in detail transaction costs, storage costs and costs associated with fulfilling a stochastic demand. In this research, we explore how exogenously determined random shocks in procurement costs affect operating decisions of a firm. In particular, we explore in detail the procurement process of commodities whose prices are subject to random shocks due to demand and supply fluctuations. Commodity prices exhibit volatility and substantial cyclical behavior that exacerbates the complexity of procurement for the commodity users. In this research, we develop optimal and approximate procurement policies of a commodity under stochastic prices and random demand. This research is motivated by crude oil procurement decisions faced by oil companies in their refinery operations. In a refinery, a sequence of complex operations are undertaken before crude oil gets refined to the desired petroleum products. Distillation of crude yields several by-products, the lightest fractions are *gases* that chiefly consists of ethane, propane and butane that are used either as fuel or as petrochemical feedstock. Lighter distillates comprise motor and aviation *gasoline* while heavier fractions yield *kerosene, gas oil and residue oil.* Residue oil is further used to produce lubricating oils, waxes and bitumen Shell (1983) (pg 244-245). Crude oil is a mix of different hydrocarbon compounds, and the proportion of the various hydrocarbons in the mix varies according to the origin of the crude oil. Table 3.1 illustrates this fact. Moreover, even for each type of crude oil the actual proportions of the different hydrocarbons can marginally vary from reservoir to reservoir.

Crude Type	Gases	Gasoline	Kerosines	Gas oil	Residue	Sulphur
Alaskan	0.8	13.4	11.6	21.5	53.3	1%
Saudi Arabian	1.7	20.5	12	21.1	45.1	3%
West Europe	4.3	22.5	12.2	21.9	39.5	0.3%
Venezuela	-	1.4	3.6	14.7	80.8	2.7%
Nigeria	2.9	25.8	14.4	12.9	29.4	0.1%
Russia	2.2	20.9	14.7	19.5	43.1	1.5%

Table 3.1: Yields for Different Type of Crude Oil Percentage by Volume

For a refinery operator, technically it is feasible to produce any refined product from any type of crude, however, it may not be economically feasible to do so. Thus, refiners often blend different types of crude in order to economically satisfy the demand for various refined products Jones (1995). Processing of crude oil begins when oil tankers bring crude oil to the docking station of an oil terminal from where oil is pumped into storage tanks. Typically, refiners have dedicated storage tanks for each type of crude. From these storage tanks oil is fed to charging tanks where different types of crudes are mixed in a proportion that depends on the actual hydrocarbon content mix of each type of crude oil in storage and on the desired mix of the refined products. This pre-mixed crude is fed from the charging tanks to cracking and distillation units for the distillation process. This is illustrated in Figure 3.1. Furthermore, market variations in the demand for refined products induce additional fluctuations in the production schedule of a refinery Paolucci et al. (2002), which affects the blending operations, therefore, creating an uncertainty about the requirements of a particular type of crude oil. Significantly, in this research we develop optimal inventory policies applicable to each particular type of crude oil which faces uncertain demand requirements, and it is also subject to fluctuations in prices as dictated by the oil market.

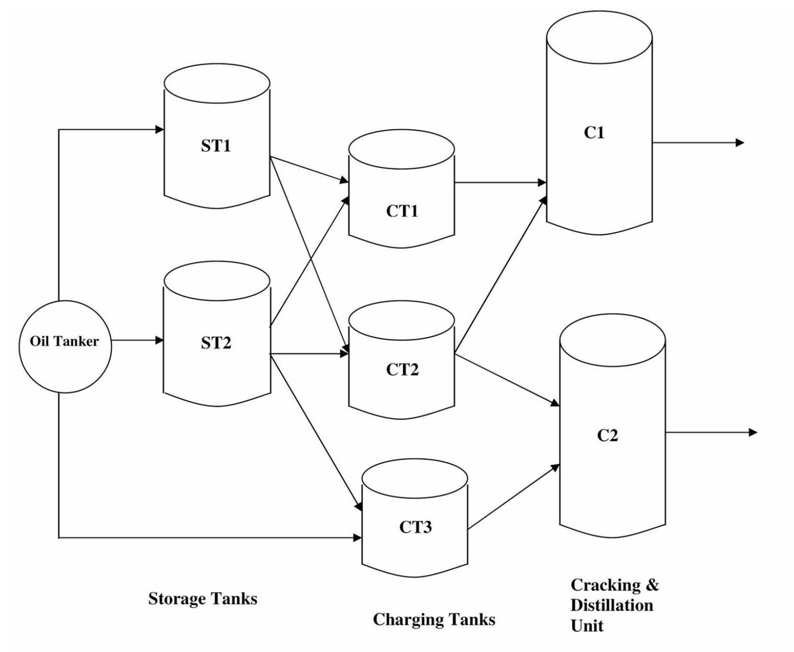

Figure 3.1: Description of Refinery Operation

Another application of our research is in the management of inventories in the jet fuel supply chain. Some commercial airlines, unable to manage the fluctuations in the price of jet fuel, have outsourced the procurement process to companies with expertise in commodities trading and hedging. For example, United Airlines has outsourced the procurement process of jet fuel to Morgan Stanley Davis (2005). Morgan Stanley, an investment bank, has expertise in the trading of commodity futures contracts, but has not traditionally dealt with the physical delivery of commodities. Recently it has been making investments in storage facilities, pipelines and tankers to facilitate efficient physical distribution and delivery of jet fuel. To effectively manage the physical distribution of jet fuel, Morgan Stanley will need to confront a number of supply chain and logistics issues that as an investment bank it did not need to confront in the past. It is increasingly becoming evident that the efficient procurement process of commodities requires the understanding of spot and futures markets in addition to the know how of supply chain management. Significantly, in this research we model the procurement decisions of a firm that operates in an environment where it deals with spot procurements and also requires the knowledge of forward markets to satisfy random demand.

Applications of this research are not limited to the above two examples as commodities have been traded in organized markets for couple of centuries now. Chicago Board of Trade (CBOT) is the prime market to trade agricultural commodities such as wheat, oat, and corn. Similarly, New York Mercantile Exchange (NYMEX) specializes in trading of crude oil and related commodities. London Metal Exchange (LME) is one of the prominent marketplace to trade metals such as copper, aluminum, zinc etc. In addition, in the recent past we have witnessed the emergence of several other B2B marketplaces for industrial commodities. These include Converge and DRAM-Exchange for semiconductors, E-steel for steel, and

Inter-continental paper exchange for paper related products. Our model, in principle, is applicable to manage the procurement process of any commodity that has an organized market.

The mathematical model developed in this chapter has an objective to model the effects of spot and futures price variability on inventory management/procurement decision making process in a multi period framework. We model the refiner as a price taker, and model commodity prices using a continuous time stochastic process that offers no risk free arbitrage opportunities. The refiners decision making process is modeled through a discrete time stochastic dynamic program that samples price information from the continuous time price stochastic process at each decision making epoch. In practice, companies make ad hoc decisions regarding spot market procurement, and to our knowledge, no inventory management model has been proposed to integrate commodity markets (that deal with spot and forward contract for storable commodities) in the procurement decisions of the firm.

Although all firms are price takers in the commodity market, the term structure of the futures price affect the internal cost of commodity users. Hence, it is judicious on part of the commodity users to adjust operating policies in lieu of the changes in spot and futures prices as reflected on the commodity markets. The concept of marginal convenience yield, as discussed in detail in the following section, provides the link between the term structure of the futures price and the effective cost of holding a commodity. In this research, we model the transaction cost associated with the physical flow of goods, and incorporate the concept of marginal convenience yield to illustrate the gains in efficiency which amounts to the reduction in inventory related costs to around 20%. This highlights the importance of this research that seeks to explore the value of including term structure of futures prices in the effective cost management policies.

3.2 Stochastic Price Model for Storable Commodities

In order to keep the exposition self-contained, in this section, we briefly introduce the literature on evolution of spot and futures commodity prices, we introduce and discuss the definition of marginal convenience yield, and we introduce the two-factor stochastic price model of Schwartz and Smith (2000) that models evolution of spot and future prices. We denote S_t as the spot price of the commodity at time t, and $E[S_{t+1}]$ as the expected spot price at time $t+1$ evaluated at time t. We define the unit holding cost per period as h, and the one-period discount factor as β.

3.2.1 Determination of Futures Prices for Storable Commodities.

Keynes (1930) proposed a theory of futures market equilibrium that allowed for a difference or bias between futures prices observed at period t for a future period $\tau + t$, $f_{t,t+\tau}$, and the expected spot price at a future period $t+\tau$ evaluated at time t, $E[S_{t+\tau}]$. Keynes argued that this bias[1] was necessary as a profit for speculators to hold risk for the processors who are willing to sacrifice some amount of expected profit in order to minimize their risk. On the other hand, Samuelson (1965) reasoned that futures prices are obtained by the current expectations of the spot price in the corresponding future period, and postulated that the futures price is an unbiased estimator of the expected spot price, $E[S_{t+\tau}] = f_{t,t+\tau}$. Dusak (1973) and Marcus (1984), through empirical studies, did not find any bias in the futures and expected spot prices for commodities. However, the empirical work of Houthakker (1957), Fama and French (1987), and Bessembinder and Chan (1992) found the existence of

[1]Specifically, Keynes argued that $f_{t,t+\tau} < E[s_{t+\tau}]$, and estimated this difference to be about 10% per year for agricultural commodities.

risk premium in financial and commodity markets. Our model allows both biased and unbiased futures prices, and our structural results are not contingent on futures price bias. The important difference is related to the implementation of our model and results; as we will discuss in section 3.4, If the commodity market shows a bias between futures and expected spot prices, a statistical analysis of the stochastic evolution of prices is necessary. On the other hand, if the market shows no such bias, all necessary information is directly observable from the commodity market's prices.

3.2.2 Convenience Yield of Storable Industrial Commodities.

At a period t, we can compare the cost of buying and storing a unit of an industrial commodity with the present value of the cost of buying it in the futures market for delivery next period. To this effect define $\hat{\delta} = S_t + h - \beta f_{t,t+1}$. As discussed in Williams and Wright (1991) $\hat{\delta} \geq 0$ and it determines the true cost of holding a unit in inventory. To quote Pindyck (2001):

> "... although the price of storage is not directly observed, it can be determined from the spread between futures and spot prices. This price of storage is equal to the marginal value of storage, i.e., the flow of benefits to inventory holders from a marginal unit of inventory, and it is termed the *marginal convenience yield*."

Consistent with Pindyck (2001), we call the quantity $\hat{\delta}$ the marginal convenience yield of holding the commodity, and it is interpreted as a premium paid for holding the commodity physically over the cost of owning a contract for future delivery. The reason for this premium is that holding the commodity physically will certainly give the holder the option to have it next period, but it may also present more profitable opportunities or uses before time $t+1$.

According to Pindyck (2001), for the aggregate market the benefit of holding inventory equals the effective cost of holding it. Moreover, the process that generates the benefits of holding inventory is not observable at the market level, but the cost of holding inventory can be observed through the spread between spot and futures prices. The cost benefit equivalence stated by Pindyck does not automatically hold true internally for an individual firm, thus, to achieve this equilibrium internally in the firm it is required to adjust the operating policies in response to the fluctuations in the convenience yield dictated by the market. Each individual firm will have a different marginal benefit of holding inventory based on its operating characteristics and its own business environment, however, the cost of holding inventory is imposed exogenously by the evolution of prices in the market.

To summarize, one of the main objectives of this research is to understand how convenience yield information, as implied by the evolution of spot and futures prices, affects optimal procurement policies for a commodity user. From the perspective of a commodity user the value of this premium is endogenously generated by his own business and process characteristics but the effective cost of holding inventories $\hat{\delta}_t$, changes randomly from period to period and it is determined exogenously by the evolution of spot and futures market prices.

3.2.3 Schwartz and Smith (2000) Two Factor Model

The two factor model of commodity pricing developed by Schwartz and Smith (2000) allows for short-term variation in prices, and uncertainty in the long term level of prices. The short term deviation in prices is defined as the difference between the current price and the long term price. These deviations may arise due to the inherent shocks in demand and supply, and the inability of the market participants to quickly adapt to the evolving market conditions. In Schwartz and Smith (2000) model, the price at time t is denoted as S_t, and it is explained as the combination of two effects: the short-term deviation factor χ_t, and the long-term equilibrium factor ω_t, such that $ln(S_t) = \chi_t + \omega_t$. The short-term deviations χ_t are assumed to revert toward zero following an Ornstein-Uhlenbeck process

$$d\chi_t \quad = \quad -\kappa\chi_t dt + \sigma_\chi dZ \tag{3.1}$$

and the long term factor is assumed to evolve randomly according to a Geometric Brownian Motion with drift factor reflecting the prospects of future supply, technological advancements, inflationary pressures and regulatory considerations, as

$$d\omega_t \quad = \quad \mu dt + \sigma_\omega dW \tag{3.2}$$

where dZ and dW are the increments of Brownian Motion that are related as $dZ.dW = \rho dt$. The parameter κ is the rate of mean reversion in the short term deviations, σ_χ is the volatility associated with the short term deviation factor, σ_ω is the volatility associated with the changes in the long term equilibrium factor. Here, it is important to understand the relationship between the spot price at time t, S_t, the expected spot price next period $E[S_{t+1}]$, and the futures price for the commodity observed at time t for delivery at $t+1$, f_t. The difference between futures price and the expected spot price can be explained as the risk-premium of the futures contract while the current price and the expected spot price next period are related by the marginal convenience yield. Below we elaborate on these two concepts.

3.2.4 Risk Premium

The futures price for period $t+1$ observed at time t, denoted as f_t can be obtained as $f_t = E^{\mathbb{Q}}[S_{t+1}]$, where the superscript $\mathbb{Q}$ is used to denote that the expectation is taken using the risk-neutral probability measure, Harrison and Kreps (1979) . Similarly, the expected spot price at period $t+1$ evaluated in period t, denoted as $\hat{f}_t$ is obtained as $\hat{f}_t = E^{\mathbb{P}}[S_{t+1}]$, where superscript $\mathbb{P}$ is used to denote that the expectation is taken using the historical probability measure. The relationship between f_t and $\hat{f}_t$ is given by $\hat{f}_t = f_t + \vartheta_t$, where ϑ_t is the risk premium associated with holding the commodity futures contract from period t to $t+1$. Schwartz and Smith (2000), show that the expected spot price at time T is evaluated at time t as

$$\begin{aligned} ln[\hat{f}_t] = ln[E^{\mathbb{P}}(S_T)] &= e^{-\kappa(T-t)}\chi_t + \omega_t + \mu(T-t) + (1-e^{-2\kappa(T-t)})\frac{\sigma_\chi^2}{4\kappa} \\ &+\frac{\sigma_\omega^2(T-t)}{2} + \frac{(1-e^{-\kappa(T-t)})\rho\sigma_\chi\sigma_\omega}{\kappa} \end{aligned} \quad (3.3)$$

On the other hand, the futures price observed at time t for the contract at matures at time T is given as

$$\begin{aligned} ln[f_t] &= ln[E^{\mathbb{Q}}(S_T)] = e^{-\kappa(T-t)}\chi_t + \omega_t + \mu(T-t) - \lambda_\omega(T-t) \\ &+(1-e^{-2\kappa(T-t)})\frac{\sigma_\chi^2}{4\kappa} + \frac{\sigma_\omega^2(T-t)}{2} + \frac{(1-e^{-\kappa(T-t)})\rho\sigma_\chi\sigma_\omega}{\kappa} \\ &-(1-e^{-\kappa(T-t)})\frac{\lambda_\chi}{\kappa} \end{aligned} \quad (3.4)$$

From (3.3) and (3.4) the risk premium is equal to $\lambda_\omega(T-t)+(1-e^{-\kappa(T-t)})\frac{\lambda_\chi}{\kappa}$. If the maturity date of the futures contracts is distant then $(1-e^{-\kappa(T-t)})\frac{\lambda_\chi}{\kappa}$ tends towards zero and the risk premium in the commodity price is exclusively dominated by λ_ω, the risk premium due to long term deviations. Also, if the mean reverting factor κ is large, it reduces the impact of λ_χ, short-term risk premium on the overall risk premium of the commodity. There is an ongoing debate in the finance and economics literature about the risk premium in commodities. Keynes (1930) argued that there is a positive risk premium for holding commodities as hedgers transfer risk to speculators and in return speculators demand risk premium. Risk premium may not exist over a long time period horizon, but it may exist in short term. This model allows computation of risk premium for short time periods as well.

3.3 Mathematical Model

In our modeling framework we consider a periodic review inventory model with stochastic demand in which there are two modes of procurement available to a risk-neutral firm, the traditional mode of procurement, in which the manufacturer enters into a contract with a supplier through a forward contract agreeing to buy a number of units to be delivered at a point in the future at set price, and the second mode of procurement which consists of trading in a commodity spot market (or alternatively, buying the commodity from a third party at spot market prices). Additionally, the manufacturer can also utilize the commodity market to sell excess inventory. Moreover, we assume that the units of the commodity bought in the spot market are delivered immediately after they are ordered, but before demand is observed, whereas the units bought through a forward contract at the beginning of the period are to be delivered at the beginning of the next decision making period. The inventory sold in the spot market is withdrawn immediately, before any customer demand is observed. We also allow for the sale of inventory through a forward contract; in this case the inventory sold shall be made available to the buyer by the beginning of the next period. In this model, we consider a finite number of decision making periods; at the beginning of each decision period, the procurement manager needs to ascertain the amount of the commodity he will buy or sell in the spot market, as well as the amount he will buy or sell using a forward contract. For simplicity, in this research we will refer to forward contracts executed at futures prices simply as futures market transactions; although technically the two transactions are different[2], for our modeling purposes they are equivalent. These spot and futures trading decisions are made at the beginning of each period (before demand is realized) with the objective of minimizing procurement, inventory holding and backlogging cost. Procurement decisions are based on the initial inventory, the spot market price, the marginal convenience yield reflected by the market spot and futures price, and the anticipated demand.

The inventory manager first estimates future demand requirements, then places orders for future delivery through normal supply channels, finally the presence of the spot market creates additional opportunity to adjust inventories before the selling period starts. The decision problem introduced above can arise in two different ways; the inventory manager may trade directly in the commodity market, or alternatively he can engage in spot and forward transactions with his traditional supplier or a third-party logistics provider. In the second case, our modeling framework still needs the existence of a commodity market to generate the spot and futures prices even though the manufacturer will not buy or sell any physical goods in this market, and we further assume that all spot and forward transactions are executed at market prices.

We next enumerate the assumptions made in this model, and discuss their

[2] Forward contracts are not traded on an exchange, and are settled by delivering the commodity, whereas futures contracts are traded on an exchange and can be settled by offsetting the current position (long or short) by taking an exact opposite position. Forward prices are equivalent to futures prices as long as interest rates are constant (Hull 2003).

relevance. First, in section 3.3.1 we assume that transaction costs are negligible. This is an assumption that has been widely made in the finance literature, and it is appropriate when we are only concerned with executing financial transactions to take or to close long or short market positions. In this model we define transaction costs more broadly to include also transportation costs, handling costs and other clerical costs associated with taking delivery from or delivering to a commodity market. This simplified model, described in section 3.3.1, will assist us in gaining preliminary insights on the trade-off between storage costs, backlogging costs and marginal convenience yield. Finally in section 3.3.2 we consider the case in which significant transaction costs are associated with both spot and forward transactions. In this case we further assume that transaction costs in the forward market are smaller than in the spot market. This assumption is justified because the lead time associated with forward market transactions allow for more cost efficient scheduling and contracting of transportation. Throughout the paper we assume that unsatisfied demand is backlogged and that each unit of the finished product requires one unit of the commodity.

The cost of holding a unit in inventory over a review period is h, and the cost of backlogging one unit of finished product for one period is denoted by p. Demand is considered to be non-negative and stochastic in each period t, and its probability mass function is denoted by ϕ_t. The distribution of demand $\phi_t(\xi_t)$ is independent of S_t. Operationally, it means that the demand ξ_t of the final product does not get affected by the spot price of the commodity; in the short-term the requirements of the commodity are inelastic relative to price. As our model is concerned with short-term procurement decisions, this is a reasonable assumption. The expected holding and penalty cost for any on hand inventory z_t is given by

$$L_t(z_t) = \int_0^{z_t} h(z_t - \xi)\phi_t(\xi)\, d\xi + \int_{z_t}^{\infty} p(\xi - z_t)\phi_t(\xi)\, d\xi$$

The derivative of the above loss function with respect to z_t is $\frac{\partial L_t(z_t)}{\partial z_t} = -p + (p + h)\Phi_t(z_t)$, where $\Phi_t(\cdot)$ is the probability distribution function of the demand.

3.3.1 Optimal Procurement Policy with no Transaction Costs.

With the purpose of studying the fundamental effects of a commodity market on optimal procurement policies we assume in this section that there are no transaction costs in spot market procurement. A stochastic dynamic program formulation is developed to identify policies that minimize expected holding cost, penalty cost, and procurement cost. The three state variables of the dynamic program are the current inventory level x_t, the short-term price factor χ_t, and the long-term price factor ω_t, respectively. The decision variable is the inventory level z_t after trading in the spot market. Any spot purchase made at time t must arrive before the demand at time t is observed. The discount factor β is defined as $\beta = e^{-r\Delta t}$, where Δt is the time interval between consecutive periodic review periods, and r is the annual interest rate. The cost to go function is calculated under a risk neutral measure, thus, we can discount the cost to go function at a risk free rate. A mathematical formulation of the optimal cost function is given by

$$V_t(x_t, \chi_t, \omega_t) = \min_{z_t} \Big\{ s_t(z_t - x_t) + L_t(z_t) \tag{3.5}$$
$$+\beta \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} V_{t+1}(z_t - \xi_t, \chi, \omega) \;\; \phi_t(\xi) \; \psi_t(\chi, \omega|\chi_t, \omega_t) \;\; d\xi \; d\chi \; d\omega \Big\}$$

and,

$$V_T(x_T, \chi_T, \omega_T) \quad = \quad -S_T x_T$$

where, $S_t = exp(\chi_t + \omega_t)$. Although S_t is a function of χ_t and ω_t for notational simplicity we omit it.

In what follows,

$$E_\xi E^{\mathbb{Q}}_{(\chi,\omega|\chi_t,\omega_t)} V_{t+1}(z_t - \xi_t, \chi, \omega) = \int_0^\infty \int_0^\infty \int_0^\infty V_{t+1}(z_t - \xi_t, \chi, \omega)\phi_t(\xi)\,\psi_t(\chi, \omega|\chi_t, \omega_t)\; d\chi\, d\omega\, d\xi \quad . \tag{3.6}$$

The triple integral in (3.6) is to be interpreted as the expected value over the demand distribution $\phi_t(\xi_t)$ and the Riemann integral over conditional joint distribution of $(\chi_{t+\Delta t}, \omega_{t+\Delta t})$ given (χ_t, ω_t) denoted as $\psi_t(\chi, \omega|\chi_t, \omega_t)$, where ψ_t[3] is the historical probability distribution implied by the two factor stochastic process of price described in section 3.2.1

Lemma 3.1. Characterization of V_t.

The cost function $V_t(x_t, \chi_t, \omega_t)$ is linear in x_t, and $\frac{\partial}{\partial x_t} V_t = -S_t$.

Proof: Immediate by defining $J_t(z_t, \chi_t, \omega_t)$ as

$$\begin{aligned} J_t(z_t, \chi_t, \omega_t) \quad = \quad & S_t z_t + L_t(z_t) + \\ & \beta \int_0^\infty \int_0^\infty \int_0^\infty V_{t+1}(z_t - \xi_t, \chi, \omega)\phi_t(\xi)\;\psi_t(\chi, \omega|\chi_t, \omega_t)\;\; d\xi\; d\chi\; d\omega, \end{aligned} \tag{3.7}$$

hence, $V_t(x_t, \chi_t, \omega_t) = \min\{J_t(z_t, \chi_t, \omega_t)\} - s_t x_t \diamondsuit$.

Next we characterize optimal procurement policies, and draw some managerial im-

[3] $\psi_t(\chi, \omega|\chi_t, \omega_t)$ is the probability density that the long term price factor and short term price factor in time $t+1$, assume the values (χ, ω) given that their values at time t are (χ_t, ω_t).

plications.

Theorem 3.1. Characterization of Optimal Procurement Policies.
The optimal policy for procurement from spot market is characterized by a point z_t^* *such that if* $x_t \leq z_t^*$ *then we buy* $z_t^* - x_t$ *commodity units from the spot market, otherwise we sell* $x_t - z_t^*$ *units in the spot market.*

Proof: The derivative of $J_t(z_t, \chi_t, \omega_t)$ with respect to z_t is given by

$$\frac{\partial J_t(z_t, \chi_t, \omega_t)}{\partial z_t} = S_t + \frac{\partial L_t(z_t)}{\partial z_t} + \beta E_\xi E^{\mathbb{Q}}_{(\chi,\omega|\chi_t,\omega_t)} \frac{\partial V_{t+1}(z_t - \xi_t, \chi, \omega)}{\partial z_t} \tag{3.8}$$

From Lemma 3.1 we know that $\frac{\partial}{\partial z_t} V_{t+1}(z_t - \xi_t, \chi, \omega) = -S_{t+1}$. Thus, it follows from the definition of $\psi_t(\chi, \omega|\chi_t, \omega_t)$ and from the definition of risk neutral measure that $E^{\mathbb{Q}}_{(\chi,\omega|\chi_t,\omega_t)} S_{t+1} = f_t$, hence the optimal z_t satisfies

$$\frac{\partial J_t(z_t, \chi_t, \omega_t)}{\partial z_t} = S_t - p + (p+h)\Phi_t(z_t) - \beta f_t = 0. \tag{3.9}$$

Denoting as z_t^* the value of z_t that solves the above equation, we get

$$\Phi_t(z_t^*) = \frac{\beta f_t - S_t + p}{p+h}. \tag{3.10}$$

♢

It is worth noting that the above optimal procurement policy is myopic in nature, and it can be characterized as a *critical-point* in the sense that if the inventory level is higher than the critical point, z_t^*, then we should sell the excess inventory in the spot market, but if the inventory level is lower than the critical point, z_t^*, we should buy the difference between z_t^* and the current inventory level from the spot market. We will refer to the ratio in (3.10) as a *critical ratio,* and it can be interpreted as follows, If we buy a unit and hold it, we pay a premium for overstocking this extra unit and is given by

$c_o = S_t + h - \beta E(S_{t+1}) = S_t + h - \beta f_t$

This can be interpreted as the cost of buying the unit at a price s_t, then holding it for the next period at a cost of h, and in doing so saving an expected present value of $\beta E^{\mathbb{Q}}(S_{t+1})$ by not having to purchase it in the next period. This cost of overstocking, c_o, corresponds to Pindyck (2001) notion of *marginal convenience yield* or price of storage. Similarly, if a firm is a unit short, it needs to replenish its stock next period, which will cost it an expected value of $\beta E(S_{t+1})$ per unit to acquire, but the firm would have saved s_t by not purchasing a unit in this period. Hence, the unit cost of under stocking is given by

$c_u = p + \beta E(S_{t+1}) - S_t = p + \beta f_t - S_t$

Observe that using the above unit cost of under stocking and overstocking in a traditional news-vendor problem framework also leads to (3.10).

We assume that sales in the forward market are executed at a risk adjusted price of f_t. This obviates the purpose of buying a unit in the forward market solely for the cost management purposes in the absence of transaction costs. Thus, in section 3.3.1 we do not model procurement from the forward market, but in section 3.3.2 we model transaction costs allowing for the procurement in the forward market.

The implementation of this optimal policy is possible because the policy is myopic in nature, hence we do not need to calculate the stochastic dynamic program explicitly. However, since f_t may not be directly observable, we need to use the Kalman filter to regress the parameters of the stochastic process (3.1) and (3.2) and use (3.4) to calculate f_t. Estimating the cost advantages of using such a policy it is not simple however; in section 3.4 we will address this issue. The myopic nature of the optimal policy is a consequence of the assumption that there are no transaction costs associated with trading in the spot market. We will relax the no-transaction costs in the next section; before we do it, however, it is worth discussing the implications of the marginal convenience yield implied by market price.

Denote by $\gamma = \beta f_t - S_t$, the spread between the expected spot price in $t+1$ price and the spot price in t; the equilibrium conditions imply that $h \geq \gamma$, which ensures that the critical ratio in (3.10) is less than 1; this condition is also equivalent to ruling out risk free arbitrage opportunities. Similarly, the condition $p + \gamma \geq 0$ can be termed a *business rationality* condition, which implies that the spot price is smaller than the sum of penalty cost and the futures price. If this condition is violated then it would not be profitable for the firm to hold any inventory or to satisfy any backlog, as it would be more profitable to sell all the stocks in the spot market and wait for future periods to satisfy the accumulated backlogs.

It is interesting to notice that when $\gamma = 0$, the optimal solution described above reduces to the solution of the standard news-vendor model. However, as γ increases firms tend to stock more as expected future spot prices increase relative to the current spot prices. However, a higher market value of marginal convenience yield $\hat{\delta}_t$ indicates that current spot prices are relatively higher, elucidating either increased demand or reduced supply for the commodity. In this case the optimal stocking policy leads the firm to lower its stocks to reduce cost since overstocking is more expensive in this period. It is interesting to note that this inverse relationship between inventory and convenience yield validated empirically by Fama and French (1988) at the industry level is also valid for each firm pursuing its own individual profit maximization efforts.

While the “true cost of holding” or marginal convenience yield is determined exogenously by the commodity market prices, individual firms can use this information to adjust their optimal stocking policies. In other words, a firm can determine its value of holding an extra unit of inventory given the value of marginal convenience yield of a commodity determined exogenously by the market. Internally for the firm, each stored unit of the commodity has different value; for example, the first unit stored by the firm will very likely be used to avert a stockout, hence its value will be close to penalty cost p, but as more units are stored the value of storing each additional unit decreases. This model allows for balancing firm specific factors, p,h and Φ with market driven factors S_t and f_t in obtaining an optimal stocking policy. If each firm in an industry make stocking decision based on the above described model then arguably it will lead to a more efficient allocation of inventory across the industry. In the absence of transaction costs, if for a firm the marginal benefit of a unit stored is less than the marginal convenience yield then this unit could be sold to another firm that places higher value on its usage.

3.3.2 Optimal Procurement Policy from a Commodity Market with Spot and Forward Transaction Costs.

In this section, we consider a supply chain configuration in which the manufacturer can procure from the spot market, but she can also commit to accepting physical delivery of the commodity at a future date. In this latter case, we assume that the contracted price per unit of the commodity is equal to the futures price observed in the commodity's market. We assume that goods bought on the spot market incur higher transportation cost in comparison to the ones bought for future delivery because they allow the manufacturer less time to arrange for its delivery. Spot shipping rates often spike up squeezing out the profit margins of firms; however, firms who enter into long term shipping contracts are unaffected by the vagaries of shipping rates in spot transportation markets. These savings in transaction costs represent an incentive for the manufacturer to commit on the purchase of goods for delivery at a future date. The model in this section assumes that at each decision period there are available both, the opportunity to buy the commodity for immediate delivery from a spot market, as well as the opportunity to buy it with a forward contract that expires, and it is delivered, at the beginning of the next decision period.

We denote by x_t the initial inventory, by z_t the inventory level after trading (buying or selling) in the spot market, and by y_t the (echelon) inventory position after trading in the forward market. Let us denote the transaction cost incurred on buying and selling a unit in the forward market by α_t^b and α_t^s respectively, $\alpha_t^{bs} = \alpha_t^b + \alpha_t^s$. Similarly, we denote the transaction cost incurred on buying and selling in the spot market as λ_t^b and λ_t^s respectively, and assume that $\alpha_t^b < \lambda_t^b$ and $\alpha_t^s < \lambda_t^s$. The procurement costs are given by $\left(S_t + \lambda_t^b\right)(z_t - x_t)^+ - \left(S_t - \lambda_t^s\right)(x_t - z_t)^+ + \beta\left(f_t + \alpha_t^b\right)(y_t - z_t)^+ - \beta\left(f_t - \alpha_t^s\right)(z_t - y_t)^+$, where the first two terms correspond to the costs associated with purchasing and selling stock in the spot market, and the last two are the costs associated with purchasing and selling in the forward market. Applying the identity $a = a^+ + (-a)^+$ these costs can be written as $\left(S_t + \lambda_t^b\right)(z_t - x_t) + \lambda_t^{bs}(x_t - z_t)^+ + \beta\left(f_t + \alpha_t^b\right)(y_t - z_t) + \beta\alpha_t^{bs}(z_t - y_t)^+$. Let $V_t^{SF}(x_t, \chi_t, \omega_t)$ denote the minimum expected procurement, shortage and storage costs from period t through period T if we can procure from spot and forward markets; then $V_t^{SF}(x_t, \chi_t, \omega_t)$ can be written as

$$V_t^{SF}(x_t,\chi_t,\omega_t) = \min_{z_t,y_t}\Big\{ \left(S_t+\lambda_t^b\right)(z_t-x_t)+\beta\left(f_t+\alpha_t^b\right)(y_t-z_t) + L_t(z_t)+\lambda_t^{bs}(x_t-z_t)^+ + \beta\alpha_t^{bs}(z_t-y_t)^+ + \beta H_t^{SF}(y_t)\Big\}, \tag{3.11}$$

$$H_t^{SF}(y_t) = \int_0^\infty\int_0^\infty\int_0^\infty V_{t+1}^{SF}(y_t-\xi_t,\chi,\omega)\ \ \phi_t(\xi)\ \psi_t(\chi,\omega|\chi_t,\omega_t)\ \ d\xi\ d\chi\ d\omega,$$

and,

$$V_T^{SF}(x_T,\chi_T,\omega_T) = (S_T+\lambda_T^b)(-x_T)^+ - (S_T-\lambda_T^s)(x_T)^+.$$

Define the function $J_t^{SF}(x_t,z_t,y_t)$ as

$$J_t^{SF}(x_t,z_t,y_t) = \left(S_t+\lambda_t^b\right)(z_t-x_t)+\beta\left(f_t+\alpha_t^b\right)(y_t-z_t) + L_t(z_t)+\lambda_t^{bs}(x_t-z_t)^+ + \beta\alpha_t^{bs}(z_t-y_t)^+ + \beta H_t^{SF}(y_t). \tag{3.12}$$

Although J_t^{SF} is convex, it can be observed from (3.12), that it is not differentiable at $z_t = x_t$, and $z_t = y_t$, hence we need to rely on its sub-differential for its optimization. To this end denote as D^- and D^+ the left and right derivative respectively, we use $D^\pm$ as a shorthand to denote 'both the left and right derivatives respectively, and denote the sub-differential of any convex function g at x with $\partial g(x)$ defined as the set valued function $\partial g\colon x \to [D_x^- g(x), D_x^+ g(x)]$Rockafellar (1970). Then a sufficient condition for x^* to minimize $g(x)$ is that $0 \in \partial g(x^*)$. We will interpret scalar addition and positive scalar multiplication operations on sub-differentials and intervals as point-wise operations on each of its elements. Thus $a + b\partial J_t^S(z+c)$ will denote the interval $\left[a + bD_z^- J_t^S(z+c), a + bD_z^+ J_t^S(z+c)\right]$ for any scalars $a, b > 0$, and c. We also define the indicator function $1_{\{A\}}$ as assuming a value of 1 whenever the logical condition$\{A\}$ is true and zero otherwise.

Lemma 3.2. Characterization of V_t^{SF} and J_t^{SF}.

The cost function V_t^{SF} is convex in x_t, and J_t^{SF} is convex in (x_t, z_t, y_t).

Proof: Immediate by induction.

In order to describe in more detail the optimal procurement policy, below we obtain the sub-differentials of J_t^{SF} with respect to z and y evaluated at (z_t, y_t), denoted as $\partial_z J_t^{SF}(z_t, y_t)$, and $\partial_y J_t^{SF}(z_t, y_t)$ respectively. To this end, observe from (3.11) that $D_{y_t}^{\pm} H_t^{SF}(y_t) = E_\xi E^{\mathbb{Q}}_{(\chi,\omega|\chi_t,\omega_t)} D_{y_t}^{\pm} V_{t+1}^{SF}(y_t - \xi_t, \chi, \omega)$, and $D_{y_t}^{+} H_t^{SF}(y_t) = D_{y_t}^{-} H_t^{SF}(y_t) = H_t^{SF'}(y_t)$. First observe that $H_t^{SF'}(y_t)$ is nondecreasing as a consequence of the convexity of V_t^{SF}; it follows from (3.12) that sub-differentials $\partial_y J_t^{SF}(z_t, y_t)$ and $\partial_z J_t^{SF}(z_t, y_t)$ can be expressed as

$$\partial_y J_t^{SF}(z_t, y_t) = \beta\left(f_t + \alpha_t^b + H_t^{SF'}(y_t)\right) + \beta\alpha_t^{bs}\left[-\mathbf{1}_{\{y_t \le z_t\}}, -\mathbf{1}_{\{y_t < z_t\}}\right], \quad (3.13)$$

and

$$\partial_z J_t^{SF}(z_t, y_t) = K_t(z_t) + \left[\beta\alpha_t^{bs}\mathbf{1}_{\{z_t > y_t\}} - \lambda_t^{bs}\mathbf{1}_{\{z_t \le x_t\}}, \beta\alpha_t^{bs}\mathbf{1}_{\{z_t \ge y_t\}} - \lambda_t^{bs}\mathbf{1}_{\{z_t < x_t\}}\right] \quad (3.14)$$

where

$$K_t(z_t) = \left(S_t + \lambda_t^b\right) - \beta\left(f_t + \alpha_t^b\right) - p + (p+h)\Phi_t(z_t).$$

Observe that $K_t(z_t)$ in a nondecreasing function, with $K_t(z_t) \to \left(S_t + \lambda_t^b\right) - \beta\left(f_t + \alpha_t^b\right) + h > 0$ as $z_t \to +\infty$; the positive value of the above limit is guaranteed by the non-negativity of the marginal convenience yield. Theorem 3.2 below characterizes an integrated forward and spot market optimal procurement policy.

Theorem 3.2. Characterization of Optimal Spot and Forward Procurement Policies.

An optimal forward procurement policy is characterized by two points y_t^b and y_t^s, $y_t^b \leq y_t^s$ as follows:

(a) ***Buy Forward:*** *If $z_t < y_t^b$, $y_t^* = y_t^b$; buy $(y_t^b - z_t)$ units.*

(b) ***Sell Forward:*** *If $z_t > y_t^s$, $y_t^* = z_t^s$; sell $(z_t - y_t^s)$ units.*

(c) ***Do Nothing:*** *If $y_t^b \leq z_t \leq y_t^s$, $y_t^* = z_t$.*

An optimal spot market procurement policy in any period t can be characterized by four spot stocking levels $z_t^b, z_t^{cb}, z_t^{cs}$ and z_t^s, with $z_t^b \leq z_t^{cb} \leq z_t^{cs} \leq z_t^s$, and the two forward levels y_t^b and y_t^s, $y_t^b \leq y_t^s$ described above. Depending on the initial inventory level, x_t, the optimal spot market procurement policy is described by the following five cases:

(d) ***Buy Spot:*** *If $x_t < z_t^b$ then $z_t^* = \max\left\{z_t^b, \min\left\{z_t^{cb}, y_t^b\right\}\right\}$*

(e) ***Conditional Buy:*** *If $z_t^b \leq x_t < z_t^{cb}$ then $z_t^* = \max\left\{x_t, \min\left\{z_t^{cb}, y_t^b\right\}\right\}$.*

(f) ***Do Nothing:*** *If $z_t^{cb} \leq x_t \leq z_t^{cs}$ then $z_t^* = x_t$.*

(g) ***Conditional Sell:*** *If $z_t^{cs} < x_t \leq z_t^s$ then $z_t^* = \min\{x_t, \max\{z_t^{cs}, y_t^s\}\}$.*

(h) ***Sell Spot:*** *If $x_t > z_t^s$ then $z_t^* = \min\{z_t^s, \max\{z_t^{cs}, y_t^s\}\}$,*

and the values of $z_t^b, z_t^{cb}, z_t^{cs}$, z_t^s, y_t^b, and y_t^s are defined as the solutions of the following equations:

$$
\begin{aligned}
K_t(z_t^b) &= -\beta\alpha_t^{bs}, && (3.15)\\
K_t(z_t^{cb}) &= 0, && (3.16)\\
K_t(z_t^{cs}) &= -\beta\alpha_t^{bs} + \lambda_t^{bs}, && (3.17)\\
K_t(z_t^s) &= \lambda_t^{bs}, && (3.18)\\
H_t^{SF'}(y_t^b) &= -f_t - \alpha_t^b, && (3.19)\\
H_t^{SF'}(y_t^s) &= \alpha_t^s - f_t. && (3.20)
\end{aligned}
$$

Proof: The proof of this theorem consists of verifying case by case that $0 \in \partial_z J_t^{SF}(z_t^*, y_t^*)$ and that $0 \in \partial_y J_t^S(z_t^*, y_t^*)$ for z_t^*, and y_t^* as specified by the theorem, and it is included in Appendix A $\diamondsuit$.

Observe that the optimal policy for forward procurement is also of the *Critical Interval* form, with the critical interval defined by $\left[y_t^b, y_t^s\right]$. However, the optimal spot market procurement policy is a little bit more complex and can be described as a *Regulated Critical Interval* since the ends of the interval depend on the forward procurement policy critical interval; specifically, this regulated critical interval is given by $\left[\max\left\{z_t^b, \min\left\{z_t^{cb}, y_t^b\right\}\right\}, \min\left\{z_t^s, \max\left\{z_t^{cs}, y_t^s\right\}\right\}\right]$. Spot procurement policy hinges on the four critical points defined as $z_t^b, z_t^{cb}, z_t^{cs}$ and z_t^s. The point z_t^b corresponds to a critical level of procurement in spot market conditional that procurement is only made in spot market and not in forward market. The z_t^{cb} is the critical level of procurement conditional that procurement is made both from spot and forward market. The point z_t^{cb} is greater than the point z_t^b because if the optimal policy of procurement in spot market is conditional that there is no procurement in forward market, as characterized by z_t^b, buying excess stock in spot market can lead to selling in forward market costing additional transaction cost. However, if the optimal procurement policy in spot market is conditional that there is procurement in forward market, the probability of selling the extra units, which are bought this period in spot market, in the forward market is very low. This reduced risk of selling in forward market potentially saves on selling transaction costs in forward market leading to more procurement in spot market. Similarly, the point z_t^{cs} corresponds to the selling point in the spot market when commodity is sold in spot and forward markets. The point z_t^s corresponds to the selling point in the spot market when commodity is sold in spot but not in forward market. Similarly, the point z_t^{cs} is less than the point z_t^s because if the commodity is to be sold in forward market it reduces the risk of selling excessively in spot market and later incurring transaction cost to buy in forward market.

Theorem 3.2(d) postulates that spot procurement is made up to z_t^b if $y_t^b \leq z_t^b$. As y_t^b increases, the lower limit of the spot procurement interval will start tracking y_t^b from z_t^b up to z_t^{cb}, as illustrated in examples (1) through (3) in Figure 3.2(a); Theorem 3.2(e) suggests that you only buy in spot market if $y_t^b > x_t$. Theorem 3.2(f) results in no procurement from the spot market since the inventory level is higher than optimal buying level z_t^{cb}, and the inventory level is lower than optimal selling point z_t^{cs}. Theorem 3.2(g) suggests that commodity is sold in the spot market if $y_t^s \leq x_t$. As y_t^s decreases, the upper limit of the critical interval will track y_t^s downward starting from z_t^s going down to z_t^{cs}, as illustrated in examples (4) through (6) in Figure 3.2(b). Similar, argument follow for Theorem 3.2(i). The optimal spot market procurement decision consists of buying/selling enough stock to get the inventory level to the interval, the difference in this case is that the end points of the interval are regulated by the forward market policy interval. After we have procured from the spot market according to the above policy, we will then procure in the forward market to bring the inventory position from z_t^* up or down to the forward policy critical interval, thus determining y_t^*.

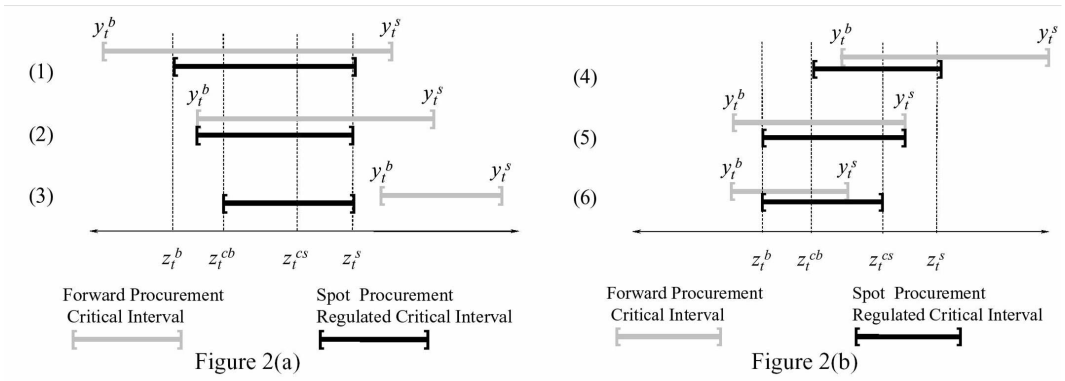

Figure 3.2: Optimal Forward and Spot Procurement Policies

Optimal policies characterized in spot and forward market are complex in nature. Although we characterize the optimal policies, it is difficult to numerically calculate y_t^b and y_t^s. Thus, we proceed to obtain lower bound on the cost function in order to approximate operating policies that are further shown to have a small optimality gap. If the transaction costs are assumed to be zero then model in Section 3.3.2 reduces to the model in Section 3.3.1, in addition, it leaves with no incentive to procure in the forward market as risk adjusted futures price is equivalent to spot price next period.

Lower Bound on V_t^{SF}. Although the values of four spot stocking levels $z_t^b, z_t^{cb}, z_t^{cs}$ and z_t^s can be easily obtained, it is very difficult to calculate y_t^b and y_t^s because they depend on $H_t^{SF}(y)$. Therefore, we proceed to obtain approximations on this policy. To this end, we can obtain a lower bound on the cost function V_t^{SF}, denoted as $\underline{\mathrm{V}}_t^{SF}$ by ignoring the term $(\beta\alpha_t^{bs})(z_t - y_t)^+ \geq 0$ in (3.11) resulting on

$$\underline{\mathrm{V}}_t^{SF}(x_t, \chi_t, \omega_t) \;=\; \min_{z_t}\left\{\underline{\mathrm{J}}_t^{(1)}(z_t, x_t)\right\} + \min_{y_t}\left\{\underline{\mathrm{J}}_t^{(2)}(y_t)\right\} \tag{3.21}$$

where,

$$\underline{\mathrm{J}}_t^{(1)}(z_t, x_t) \;=\; \left(S_t + \lambda_t^b\right)(z_t - x_t) - \beta\left(f_t + \alpha_t^b\right) z_t + L_t(z_t) + \lambda_t^{bs}(x_t - z_t)^+ \tag{3.22}$$

$$\underline{\mathrm{J}}_t^{(2)}(y_t) \;=\; \beta\left(f_t + \alpha_t^b\right) y_t + \beta\underline{\mathrm{H}}_t^{SF}(y_t), \tag{3.23}$$

$$\underline{\mathrm{H}}_t^{SF}(y_t) \;=\; \int_0^\infty\int_0^\infty\int_0^\infty \underline{\mathrm{V}}_{t+1}^{SF}(y_t - \xi_t, \chi, \omega)\;\; \phi_t(\xi)\;\psi_t(\chi, \omega|\chi_t, \omega_t)\;\; d\xi\; d\chi\; d\omega,$$

and,

$$\underline{\mathrm{V}}_T^{SF}(x_T, \chi_T, \omega_T) \;=\; V_T^{SF}(x_T, \chi_T, \omega_T) = (S_T + \lambda_T^b)(-x_T)^+ - (S_T - \lambda_T^s)(x_T)^+. \tag{3.24}$$

Since the transaction costs of selling excess inventory in the forward market are ignored to obtain this lower bound on the cost function, the two points y_t^s and y_t^b as described in Theorem 3.2 converge to a single point y_t^*. In addition, the four points associated with the spot procurement policy as described in the Theorem 3.2 finally reduces to only two points given as $z_t^{cb}$and z_t^s. This observations are formalized in the following theorem.

Theorem 3.3: *The optimal procurement policy $\underline{z}_t^*$, y_t^* associated with $\underline{\mathrm{V}}_t^{SF}$ is characterized by the sequences z_t^{cb}, z_t^s and y_t^*, with $z_t^{cb} \leq z_t^s$ such that*

a) *if $x_t < z_t^{cb}$, then $\underline{z}_t^* = z_t^{cb}$*

b) *if $z_t^s < x_t$ then $\underline{z}_t^* = z_t^s$*

c) *if $z_t^{cb} \leq x_t \leq z_t^s$ then $\underline{z}_t^* = x_t$.*

Thereafter, the inventory position is adjusted to y_t^* by procuring/selling in the forward market.

Proof: Immediate from Theorem 3.2 by setting $\alpha_t^{bs} = 0\diamondsuit$.

At any period t the points z_t^{cb} and z_t^s can be calculated from (3.16) and (3.18) respectively. Obtaining y_t^* is more involved. First observe from (3.24) that the optimization of z_t has been decoupled from the optimization over y_t, hence the derivatives of $\underline{\mathrm{H}}_t^{SF}(y_t)$ and $\underline{\mathrm{H}}_{t+1}^{SF}(y_{t+1})$ can be obtained independently from each other. Specifically, since $x_{t+1} = y_t - \xi_t$,

$$\frac{\partial}{\partial y_t}\underline{\mathrm{H}}_t^{SF}(y_t) = \beta E_\xi E^{\mathbb{Q}}_{(\chi,\omega|\chi_t,\omega_t)} \frac{\partial}{\partial y_t}\underline{\mathrm{V}}_{t+1}^{SF}(y_t - \xi, \chi, \omega) = . \tag{3.25}$$
$$\beta E_\xi E^{\mathbb{Q}}_{(\chi,\omega|\chi_t,\omega_t)} \frac{\partial}{\partial y_t} \min_{z_{t+1}} \left\{ \underline{\mathrm{J}}_{t+1}^{(1)}(z_{t+1}, y_t - \xi) \right\}$$

Even though $\frac{\partial}{\partial y_t}\underline{\mathrm{H}}_t^{SF}(y_t)$ is difficult to compute directly, it is feasible to estimate it using Monte Carlo simulation as follows. First we compute the loss function $L_{t+1}(z)$ which will not change for any (χ_t, ω_t) sample path. Then we simulate a sample path from (χ_t, ω_t) to $(\chi_{t+1}, \omega_{t+1})$, and obtain the associated values of z_{t+1}^{cb} and z_{t+1}^{s}. After obtaining the parameters defining $\underline{\mathrm{z}}_{t+1}^*$ we calculate form (3.22) the expected value $E_\xi \frac{\partial}{\partial y_t}\underline{\mathrm{J}}_{t+1}^{(1)}(\underline{\mathrm{z}}_{t+1}^*, y_t - \xi)$ for a range of values of y_t, and repeat this process for different (χ_t, ω_t) sample paths until we obtain a stable average over the range of interest of y_t. Then the value of y_t^* can be estimated by minimizing $\underline{\mathrm{J}}_t^{(2)}(y_t)$ as indicated in (3.24).

Upper Bound on V_t^{SF}. An upper bound $\bar{V}_t^{SF}$ on V_t^{SF} can be obtained by setting $\bar{V}_t^{SF}(x_t, \chi_t, \omega_t) = J_t^{SF}(x_t, z_t, y_t)$ for any values of z_t and y_t, and J_t^{SF} as defined in (3.12). Thus the values of z_t^* and y_t^* obtained for the lower bound calculation can certainly be used on J_t^{SF} to obtain also an upper bound; However, the calculation of y_t^* though feasible is time consuming, and we would like to propose an approximation $\bar{y}_t^*$ that is easier to obtain, and hence easier to implement. To this end, instead of calculating the expected value of the spot market policy in period $t+1$ we approximate it with the spot market policy obtained as if $S_{t+1} = E\left[S_{t+1}|\chi_t, \omega_t\right] = f_t$. We can then obtain $\bar{z}_{t+1}^{cb}$ and $\bar{z}_{t+1}^{s}$ from (3.16) and (3.18) thus defining a spot market policy $\bar{z}_{t+1}$ for period $t+1$ in period t. With this approximate policy for period $t+1$

we can proceed to modify (3.25) to estimate the derivative of $\underline{\mathrm{H}}_t^{SF}(y_t)$ as

$$\frac{\partial}{\partial y_t}\bar{H}_t^{SF}(y_t) = \beta E_\xi \frac{\partial}{\partial y_t}\underline{\mathrm{J}}_{t+1}^{(1)}(\bar{z}_{t+1}, y_t - \xi), \tag{3.26}$$

and then obtain the value of $\bar{y}_t^*$ by minimizing $\underline{\mathrm{J}}_t^{(2)}(y_t)$ using the value of $\frac{\partial}{\partial y_t}\bar{H}_t^{SF}(y_t)$ to approximate $\frac{\partial}{\partial y_t}\underline{\mathrm{H}}_t^{SF}(y_t)$ when calculating the derivative of $\underline{\mathrm{J}}_t^{(2)}(y_t)$. Having obtained $\bar{y}_t^*$ we define the upper bound $\bar{V}_t^{SF}(x_t, \chi_t, \omega_t) = J_t^{SF}(x_t, \underline{\mathrm{z}}_t^*, \bar{y}_t^*) \geq V_t^{SF}(x_t, \chi_t, \omega_t)$. In Section 3.4 we evaluate the gap between $\bar{V}_t^{SF}$ and $\underline{\mathrm{V}}_t^{SF}$.

3.4 Numerical Analysis

In this section we explore the benefits of incorporating marginal convenience yield information in the derivation of stocking policies over stocking policies that are obtained ignoring this information. The simplest policy we consider for benchmarking purposes is one that optimizes holding and penalty costs, but ignores the effect of price changes in the spot market, we term this as static news vendor (SNV) policy. Under the SNV stocking rule, a firm sets its stocking levels for all subsequent periods based on the price of commodity observed in the first period. A second, more elaborate policy that we will call a constant convenience yield (CCY) policy, a firm has access to commodity prices, but uses the value of the marginal convenience yield, which is an average over an entire planning horizon. This is equivalent to assuming a constant convenience yield as assumed by Berling and Rosling (2005). Then, we evaluate a dynamic convenience yield (DCY) policy, as obtained in Section 3.3.2 DCY policy captures the fluctuations in convenience yield due to fluctuations in spot and futures prices. For the purposes of comparison we calculate the expected costs over a 50 period horizon.

In order to analyze the cost structure better we break the cost into two categories, *uncontrollable* and *controllable* costs. Uncontrollable costs are the ones that can not be influenced by any stocking policy and are completely dependent on the path of the stochastic process; controllable costs, on the other hand, are those affected by stocking policy. For brevity we illustrate the cost breakdown for the model in Section 3.3.1 only; for this model the cost of a spot price and demand sample path can be written as follows:

$$\begin{aligned} C(x_t) &= \sum_{t=0}^{N} \Big[(S_t)(z_t - z_{t-1} + \xi_{t-1})^+ - (S_t)(z_{t-1} - z_t - \xi_{t-1})^+ + h(z_t - \xi_t)^+ \\ &\quad + p(\xi_t - z_t)^+ \Big], \end{aligned}$$

where $x_t = z_{t-1} - \xi_{t-1}$; through algebraic manipulation this cost function can be re-written as

$$\begin{aligned} C(x_t) &= \sum_{t=0}^{N} \left[(s_t - s_{t+1})(z_t - \xi_t) + h(z_t - \xi_t)^+ + p(\xi_t - z_t)^+ \right] \\ &\quad + \sum_{t=0}^{N} (s_t)\xi_t. \end{aligned}$$

We refer to the first three terms respectively as appreciation/depreciation of the safety stock (ADSS), holding cost, and penalty cost respectively. These first three terms together comprise controllable cost. The second summation in the above expression is referred to as the uncontrollable cost. Furthermore, under reasonable assumptions we can show that the controllable costs are non-negative, and, we will use them as the basis of comparison of the relative benefits of the DCY model. The percentage benefits of DCY over CCY policies is given by Δ_{CY-D}, and the benefit of CCY over SNV policy is given by Δ_{C-S}. These percentages have been calculated in the following way. $\Delta_{CY-D} = \frac{\Omega_{CCY}-\Omega_{DCY}}{\Omega_{CCY}-\Omega_{UC}} \times 100$, where Ω_{DCY} is the cost under DCY model, Ω_{CCY} is the cost under the CCY policy, and Ω_{UC} is the uncontrollable cost. Similarly, $\Delta_{C-S} = \frac{\Omega_{SNV}-\Omega_{CCY}}{\Omega_{SNV}-\Omega_{UC}} \times 100$, where Ω_{SNV} is the cost under static newsboy model's policy.

We estimate the value function of the dynamic programs in Section 3.4 using Monte Carlo simulation. For this purpose, we simulate 2000 sample paths of stochastic price process, and 50 demand paths for each price path, adding up to a total of 100,000 sample path replications for each set of parameters. Table 3.2 summarizes the set of base case [4]and sensitivity parameters used to quantify the magnitude of the advantage of using the CY model. This is done across ranges of the following parameters: penalty cost p, coefficient of variation of demand, volatility of short-term deviations in price σ_χ, volatility of long term deviation in price σ_ω, speed of mean reversion k, and coefficient of correlation between the two underlying Brownian Motion processes, ρ.

[4]The set of parameters we use as a Base Case are those obtained by Schwartz and Smith (2000) for oil markets.

Stochastic Process Parameters		Base case value
Drift rate	μ	-0.0125
Volatility in short-term deviations	σ_χ	0.3
Volatility in long-term deviations	σ_ω	0.15
Coefficient of mean reverting process	κ	1.5
Correlation in short and long term deviations	ρ	0.3
Operating Parameters*		
Penalty cost per unit per period	p	0.5
holding cost per unit per period	h	0.5
Std Deviation in demand	cv	10
Transaction cost of buying from the spot market	η	2.0
Transaction cost of buying from forward market	α	0.5
Initial conditions		
value of short-term deviation	χ_t	2.8
value of long-term deviation	ω_t	0.3
Spot price	S_0	22.2
General		
Review period interval	Δt	1
Brownian motion increment	dt	0.02
Planning horizon		50

Table 3.2: Stochastic Price Process Parameters

		P		SD		σχ		σω		ρ		K	
	Base Case	0.3	1	5	20	0.1	0.5	0.05	0.25	0.1	0.5	0.5	5
DCY Model													
Penalty Cost	117.33	149.33	107.89	60.94	230.12	111.36	145.3	117.97	115.38	116.94	117.69	95.61	209.56
Holding cost	59.25	33.37	103.28	29.63	118.51	56.15	65.97	58.05	61.32	58.82	59.7	65.23	68.56
ADSS	-27.52	-100.94	15.84	-16.2	-50.19	-11.28	- 85.45	-26.16	-29.02	-26.07	-29.22	-3.4	-185.9
Control. Cost	149.06	81.76	227.01	74.37	298.44	156.23	125.8	149.86	147.68	149.69	148.17	157.44	92.23
CCY													
Penalty Cost	107.6	106.88	105.23	53.88	215.53	108.76	105.2	103.34	104.84	108.18	107.34	95.43	110.41
Holding cost	55.83	29.06	100.15	27.91	111.67	55.29	57.22	54.98	57.44	55.6	56.07	62.91	54.42
ADSS	-2.4	-29.916	28.45	-1.2	-4.8	-3.08	-1.08	-3.34	-0.86	-2.65	-2.16	2.07	-1.82
Control. Cost	161.03	106.024	233.83	80.59	322.4	160.97	161.4	154.98	161.42	161.13	161.25	160.41	163.01
SNV													
Penalty Cost	69.49	64.43	70.58	34.74	138.98	69.5	69.49	69.49	69.49	69.49	69.5	69.5	69.49
Holding cost	82.54	56.05	125.48	41.27	165.1	82.54	82.54	82.54	82.54	82.54	82.54	82.5	82.54
ADSS	17.39	-2.29	42.69	8.7	34.79	18.33	15.81	17.98	16.37	17.65	17.13	11.97	20.8
Control. Cost	169.42	118.19	238.75	84.71	338.87	170.37	167.8	170.01	168.4	169.68	169.17	163.97	172.83
ΔCY_D	7.52	22.89	2.92	7.71	7.43	2.95	22.03	6.9	8.51	7.1	8.11	1.86	43.42
ΔC_S	12.02	30.85	4.92	12.2	11.93	8.3	25.04	11.83	12.31	11.79	12.42	4.01	46.66

Table 3.3: Sensitivity Analysis

3.4.1 Procurement Model with No Transaction Costs.

We observe from Table 3.3 that for the Base Case the potential gain from the DCY model over CCY is 7.52%, on the other hand CCY gains 4.87% over SNV. Benefits from DCY model over CCY model can go upto 40% depending upon the process parameters. The potential gains in the CCY and DCY model accrue from the gains on ADSS. Since stocking levels in CCY and DCY model are less than from that in SNV model, the holding cost is lower and penalty costs are higher, but due to the information on marginal convenience yield it is possible to save cost on the ADSS stock. Thus, the overall benefit on ADSS stock outweighs the increase in the combined cost of penalty and holding. *Effect of p/h Ratio.* Increase in the ratio p/h seems to dissipate the benefit of CY model because higher penalty cost augments the stocking level, which reduces the opportunity to minimize the cost of ADSS.

Effect of Demand Variability. The increase in demand variance can have a twofold impact on the cost structure, on one hand the penalty and holding cost will increase, and on the other hand savings on ADSS will also increase. However, we observe that these two effects counterbalance their influence thus, increases in demand variance does not significantly change the value of DCY model on a percentage basis, however, the dollar value of the benefit increases. *Effect of volatility of short-term deviations.* An increase in volatility of short-term deviations leads to an increase in penalty and holding costs, but creates an additional opportunity to save cost on ADSS stocks by adjusting the safety-stock level. Thus, benefits from using DCY model increases as the volatility in short-term deviation of prices increases. *Effect of volatility of long-term deviations.* Increase in fluctuations in long-term deviations does not manifest a significant change in benefits drawn from DCY model. *Effect of Correlation Coefficient:* Greater correlation between two underlying Brownian motions leads to marginally greater potential benefit from DCY model as a high correlation coefficient ensures better predictability of futures prices. *Effect of Speed of Mean Reversion :* High value of κ leads to higher benefits from the DCY model because higher κ ensures faster mean reversion in short-term deviations which in turn lowers the variance in predictability of futures price Schwartz and Smith (2000) which enhances the benefits from the DCY model. To summarize, the benefits of DCY policy over CCY policy are considerably high when volatility in short-term deviation is high and when mean reverting factor is high.

3.4.2 Model with Transaction Cost in Spot and Forward Markets

In spite of incorporating transaction costs in both the spot and forward markets, we observe a high performance of the sub-optimal policies developed in Section 3.3.2, as implied by the tightness of the bounds on the cost function. The worst case observed for the given set of parameters is given by a gap of 1.8% between lower and upper bound as shown in Table 3.4. Since in this given supply chain configuration the manufacturer accepts the physical delivery from the supplier (or from the forward market), he attempts to purchase more from the forward market availing the opportunity of lower transaction cost in the forward market. In addition, spot markets are utilized to fine tune the balance of demand and supply by operating at a relatively higher transaction cost. Table 3.5 shows that procurement in spot and forward market decreases as the transaction cost increases. Additionally, we also observe that selling in spot market decreases with increases in the transaction cost in forward market. This primarily occurs because as buying in forward markets gets more expensive, it reduces the stocking level from forward market, and thus, reduces the possibility of carrying excessive inventory into the next period.

T. Cost in Spot Market $\lambda^s_t=\lambda^b_t=\lambda=$ **T. Cost in Futures Market**	**0.1**	**0.2**	**2**	**4**
0.2*λ	0.01	0.02	0.13	0.39
0.5*λ	0.04	0.09	0.36	1.0
0.8*λ	0.1	0.25	0.68	1.8

Table 3.4: Gap Between Bounds- Spot and Forward Transaction costs

Table 3.5: Percentage Breakdown of Procurement in Spot and Forward Market

T. Cost in Spot Market $\lambda^s_t=\lambda^b_t=\lambda=$ **T. Cost in Futures Market**	**0.1**	**0.2**	**0.5**	**2**
0.2*λ				
Buying in Spot Market	38.74	34.2	15.28	1.06
Selling In spot Market	34.34	27.76	4.4	1.2
Buying in Futures Market	97.88	96.54	99.52	10.8.6
Selling in Futures Market	2.56	3.03	10.5	8.78
	100.00	**100.00**	**100.00**	**100.00**
0.5*λ				
Buying in Spot Market	43.06	39.4	30.44	1.74
Selling In spot Market	30.48	20.9	1.82	0.92
Buying in Futures Market	92.18	87.74	83.22	108.2
Selling in Futures Market	5.02	3.16	12.06	9.32
	100.00	**100.00**	**100.00**	**100.00**
0.8*λ				
Buying in Spot Market	49.9	47.1	42.22	16.8
Selling In spot Market	35.4	13.5	0.75	0.54
Buying in Futures Market	84.5	78	73.2	94
Selling in Futures Market	9.04	11.4	14.5	10.5
	100.00	**100.00**	**100.00**	**100.00**

3.5 Managerial Insights

Firms are price takers in the commodity markets, however, the term structure of the futures price influences the internal cost of operation of a commodity user. The concept of marginal convenience yield Pindyck (2001) links the term structure of future price with the effective holding cost of the commodity. In this research, we illustrate that using the information on marginal convenience yield can lead to significant reduction in inventory related cost.

The model developed in this chapter has the following contributions: a) we model stochasticity of prices in a multi-period, periodic review inventory system, b) we evaluate the procurement policy of a firm in a market paradigm, assimilating the information on marginal convenience yield in procurement decision making, and

c) we devise the optimal policy structure of procurement from spot and forward markets taking into consideration transaction costs. Commodity pricing has been studied in the literature of finance, whereas, in the operations management literature, inventory policies have been generally studied ignoring marginal convenience yield information. We attempted to bridge these two streams of research by obtaining operational decisions using commodity market information. We characterize optimal procurement policies, develop bounds and approximations to facilitate its implementation, and quantify the tightness of the bounds. Furthermore, we postulate that for optimal decision making a firm should balance its marginal profit of storing each additional unit with the marginal convenience yield determined exogenously by the spread between spot and futures market prices. If a firm's marginal profit for an additional unit is lower than the market marginal convenience yield, then the firm is better off selling that unit in the market, as some other user of that unit is willing to pay more than the value the firm places on this unit. Moreover, these trading decisions are mediated by transaction costs derived from the logistics of interacting with these markets.

Futures markets have been traditionally used for risk-hedging purposes, however, we show that information on futures prices can have additional advantages other than risk-management. Information on futures prices can be used by a commodity user to fine tune its stocking policies in order to minimize its inventory related costs. Our results illustrate that commodity markets provide flexibility in procurement to fine tune stocking levels, but they also highlight that another important benefit of commodity markets is the spot and futures price information that they provide, leading commodity users to more efficient procurement decisions.

Chapter 4

Procurement and Distribution Policies in a Distributive Supply Chain

4.1 Introduction

In the recent past there has been an upsurge in the trading of industrial commodities on established commodity exchanges. NYBOT and CBOT started trading long term futures contracts for ethanol in the year 2005. Also, NYBOT recently introduced futures and option contracts on wood pulp. London Metal Exchange (LME) started trading futures contracts on plastics, and is deliberating to start trading futures contracts on steel. Although commodities have been traded on organized markets for centuries now, the above cited examples refer to industrial commodities which highlights the recent surge in significance of commodity markets in manufacturing sector of the economy. Commodity markets provide information about price trends through prices of futures, and although the potential use of futures contracts for hedging purposes has been well established, the impact of price information on procurement and distribution policies in a supply chain is not well understood. Significantly, this research explores how the price information generated by commodity markets can be utilized by commodity users and commodity processors to improve their internal decision making.

This paper investigates the effect of fluctuating prices of commodities as dictated by commodity markets on procurement and distribution policies in a supply chain. The spread between spot and futures prices as dictated by markets determine the actual cost of holding a commodity, often referred in the economics literature as the cost of carry Working (1949) and Williams and Wright (1991). The objective of this research is to ascertain efficient procurement and distribution policies when markets are exogenously imposing the cost of holding on a firm. In this context, it becomes important to understand how a firm should adjust its operating policies in response to the dynamically changing commodity prices. In general, commodities, such as gasoline which is the motivation for this paper, are distributed through multi-echelon inventory distribution systems. In this research, we seek to explore the role of commodity markets on the procurement and distribution policies of commodities in a two-echelon distribution network.

In general, the spot price of a commodity reflects the existing dynamics of supply and demand in the market, whereas futures prices reflect the anticipated demand and supply equilibrium in the future. Moreover, the information generated by the futures markets can also be useful in negotiating forward[1] contracts between

[1]Forward contract is a customized non-transferable agreement between buyer and seller, and are settled through physical delivery. Futures contracts, on the other hand, are standardized contracts that are settled by financially offsetting an existing position (long or short), and are commonly used for risk hedging purposes.

buyers and suppliers of the commodity since the future price dictated by the market contains more information on demand/supply dynamics than the information held by individual buyers and suppliers. The objective of this paper is to understand how this price information generated by commodity markets can be utilized to enhance the efficacy of the supply chain through better procurement and distribution policies. In particular, we aspire to understand how the operational efficiency of a supply chain can be enhanced by reflecting the market generated true cost of storage on supply chain procurement and distribution decisions. The main motivation for this paper is the distribution process of gasoline in a vertically integrated oil company.

The supply chain of gasoline is a very complex network that distributes around 9 million barrels per day across 167,893 retail gas stations spread across the country. Distribution of gasoline starts from a refinery where crude oil is refined into gasoline and other products, gasoline is then shipped through a network of pipelines to various terminals (that act as distribution locations), where it is loaded on tanker trucks and shipped to the network of retailers. These terminals house storage facilities, where additives are added to gasoline according to the requirements of the various different brands as well as local regulations. There are two types of retail stations, those supplied directly by the refiner, and those that are supplied by independent wholesalers who act as intermediaries between the refiners and independent retailers. Directly supplied gas stations sell branded gasoline and operate under the following three different types of supply contracts: (a) stations that are owned and operated by the refiner, (b) stations that are owned by the refiner, but leased to an operator who manages it and has the ability to set retail prices, and (c) stations that are independently owned, and the owner has signed an agreement with the oil company to sell its brand of gas Hastings (2004). Gas stations that are independently supplied can be subdivided into those who sell branded or unbranded gasoline; they procure gasoline from jobbers operating at the terminal. Jobbers are wholesalers who buy gasoline from the terminal and distribute unbranded or branded gasoline to the retail network downstream. In our research, we focus on the procurement of gasoline at the terminal and the direct distribution of gasoline to the branded retailers. In general, all three types of directly supplied branded retailers operate under Vendor Management Inventory (VMI) policies, therefore, from a cost management perspective this system can be treated as a vertically integrated channel. The VMI system has been successfully implemented with the help of advancements in information technology; for instance, gas tanks buried under the gas station have electronic sensors that convey the inventory level to a procurement management team at the terminal, where logistical arrangements are made to efficiently deliver gasoline to each specific retail station Worthen (2002)

Gasoline at the terminal can be procured internally, through forward contracts and through spot purchases. In the wake of supply disruptions or spikes in consumer demand, inventory at the terminal may fall below desirable levels forcing terminal mangers to procure gasoline from the spot market. For instance when Phillips Petroleum ran out of gasoline at its Phoenix terminal (Barrionuevo, 2002), it temporarily closed many retail gas stations resulting in large penalty costs and high procurement costs from spot markets. According to CIO magazine Worthen (2002), up to 30% of the gasoline supply at Chevron is obtained through spot market purchases. Thus, from the interest of an operations manager some pertinent questions include 1) how much gas should be procured through spot and forward transactions at the terminal 2) what should be the distribution policy to the retailers, and 3) what is the effect on system performance of using the information generated from the price observed in commodity markets to formulate the above two procurement and distribution policies. These are the questions we seek to address in this research using a two echelon inventory model where the upper echelon in the model is analogous to the terminal.

The distribution network consists of multiple non-homogeneous retailers fulfilling random demands using a periodic review demand replenishment system. The central procurement facility, a gasoline terminal, can procure the commodity from the spot market with immediate delivery, as well as from the forward market with a delivery lag of one period. The transportation costs associated with the procurement from spot and forward markets have also been modeled. We assume that the demand process and commodity price process are independent from each other. This assumption is justified in our example as the demand for gasoline is relatively inelastic in the short term for moderate variations in price. We also assume that commodity prices follow a stochastic process that offers no risk-free arbitrage opportunities.

4.2 Serial Echelon Model

There is a range of possible models that represent the distribution of gasoline. On one end of spectrum there is a case where retailers may be identical in their penalty costs and have a stationary demand distribution. However, on the other end of the spectrum is the case where each retailer may face a different penalty costs due to stock outs and non-stationary demand distribution. In a scenario where retailers have identical penalty costs and stationary demand distribution, it has been shown by Federgruen and Zipkin (1984) that all retailers can be aggregated into one entity, and then a critical fractile allocation policy can be optimally obtained for this combined entity. This derived critical fractile can be used to determine the stocking levels at each retailer. In this section, we assume that retailers have identical penalty cost and stationary demand distribution. This section develops a methodology to obtain a characterization of the optimal solution when commodity prices are fluctuating and there is an additional possibility of procurement from the spot market. In Section 4.3 we develop a solution methodology for the case where retailers are non identical and demand is non-stationary.

This model addresses a two echelon inventory system in which a single commodity is procured at the higher echelon either through spot market or through forward contract transactions, and then this commodity is shipped to the lower echelon as shown in Figure 4.1. Specifically, in our model it means that the gasoline is procured centrally at the terminal through spot and forward market transactions and, thereafter, shipped to the retailer. Since gasoline is reformulated at the terminal, it is not economical to re-sell it in the commodity market; therefore, this system summarizes a make-to-stock manufacturing system, which implies that commodity once bought from the market and reformulated can not be resold in the commodity market. The commodity bought from the forward market arrives with a lead time of one period while spot procurement has a zero lead time. In particular, both spot and forward procurement decisions are made before the demand in the period is realized. Moreover, the transaction cost from the forward procurement α_t is considered to be lower than the spot transaction cost η_t because in spot trade the goods have to be transported with a short notice and this usually results in a higher cost of transportation. In addition, we consider a zero lead time in shipping of gasoline from the terminal to the the retailer at a transportation cost of γ_t. This system operates under periodic review system, where unsatisfied demand at the retailer is completely backlogged. The cost of each unit of unsatisfied demand is p_t in period

t , and the cost of storage is h_t per unit in period t. We denote ξ by the random demand , and $\phi_t(\xi)$ its probability density function.

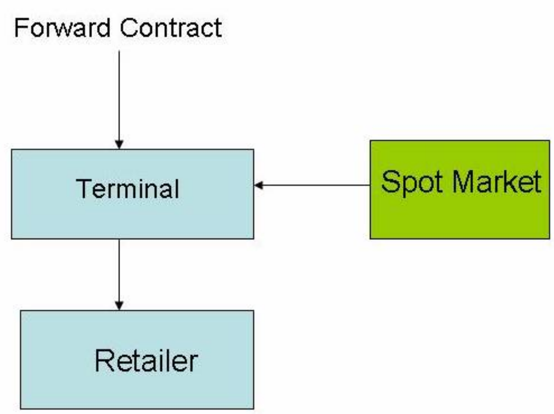

Figure 4.1: Serial Multi-Echelon Distribution System

The state of the system can be represented as follows, w_t denotes the inventory level at the retailer, and the echelon inventory at the terminal is denoted by x_t. The price of the commodity in period t is determined by the realization of χ_t and ω_t as described in Section 3.2.3, where $S_t = exp(\chi_t + \omega_t)$. At the start of each period three stocking decisions are made; first, the procurement decision from the forward market is made, we denote the inventory position as y_t which represents the echelon stock; second the spot procurement decision is made, we denote inventory level after spot transactions as z_t; finally, after procurement decisions are made, the allocation of inventory to the lower echelon is decided, we denote v_t as the inventory position at the retailer. Stocking levels at the terminal are modeled as echelon stocks with the inventory position obtained from the forward contract y_t considered to be higher than or equal to the inventory position obtained after procurement from spot market

z_t . The loss function of retailer is defined as

$$L_t(v_t) = \int_0^{v_t} h_t(v_t - \xi_t)\phi_t(\xi)\, d\xi + \int_{v_t}^{\infty} p_t(\xi - v_t)\phi_t(\xi)\, d\xi$$

The overall objective of the firm is to minimize the following cost function, which is modeled as a stochastic dynamic program whose state space is represented by short term deviation factor χ_t, long-term deviations factor ω_t, the initial echelon inventory at the terminal x_t, and initial inventory at the retailer given as w_t.

$$\begin{aligned} J_t(\chi_t, \omega_t, y_t, z_t, v_t, x_t, w_t) &= (S_t + \eta_t)(z_t - x_t) + (\beta f_t + \alpha_t)(y_t - z_t) \quad (4.1) \\ &\quad + h_t(z_t - x_t) + \gamma_t(v_t - w_t) + L(v_t) + \\ &\quad \beta E_{\chi,\omega|\chi_t,\omega_t} E_\xi V_{t+1}(\chi, \omega, y_t - \xi_t, v_t - \xi_t) \quad (4.2) \end{aligned}$$

$$V_t(\chi_t, \omega_t x_t, w_t) = \min_{y_t, z_t, v_t} \left\{ J_t(\chi_t, \omega_t, y_t, z_t, v_t, x_t, w_t) \right\} \quad (4.3)$$

where, $ln(S_t) = \chi_t + \omega_t$

s.t $\quad y_t \geq z_t \geq x_t; \quad z_t \geq v_t \geq w_t;$

and,

$$V_T(\chi_T, \omega_T, x_T, w_T) = -(S_T + \eta_T)(-x_T)^+ + (S_T - \eta_T)(x_T)^+$$

The functions J_t and V_t are a function of χ_t and ω_t, however, for notational simplicity we will omit this for the remainder of paper. The first term in (4.1) represents spot procurement costs, the second term denotes forward procurement cost, the third term accounts for holding cost on the upper echelon, the fourth term captures the transportation cost from the upper echelon to the lower echelon, the fifth term is the loss function, and the sixth term represents the cost to go function. At the end of the horizon, backlogged demand is fulfilled from spot market purchases, and excess stock is salvaged by selling it on the spot market. We consider a transaction cost of η_T per unit for selling stocks in the spot market in the last period. In what follows, we show that the methodology of Clark and Scarf (1960) in conjunction with the results proposed by Karush (1958) leads to the decomposition of multi-echelon problem in the above presented framework. The following lemma elucidates the results of Karush in more detail. In what follows, we denote $min\{a,b\} = a \wedge b$, and $max\{a,b\} = a \vee b$.

Lemma 4.1: **Karush:** *If a function f is convex on $\mathbf{R}$, and $x \leq y$, then $min_{x \leq \theta \leq y} f(\theta)$ $= f(\theta^*) + H(x) + G(y)$, where a is constant, H is convex increasing and G is convex decreasing such that*

$H(x) = f(x \vee \theta^*) - f(\theta^*)$, and

$G(y) = f(y \wedge \theta^*) - f(\theta^*)$, where θ^* is the unconstrained minimum of the function $f(\theta)$.

Theorem 4.1: Convexity and Separability of V_t

The cost function $V_t(x_t, w_t)$ is convex in (x_t, w_t), and can be decomposed as $V_t(x_t, w_t) = V_t^1(w_t) + V_t^2(x_t)$, where $V_t^1(w_t)$ is convex in w_t and $V_t^2(x_t)$ is convex in x_t.

Proof: We observe that $V_T^2(x_T) = (S_T + \eta_T)(-x_T)^+ - (S_T - \eta_T)(x_T)^+$, and $V_T^1(w_T) = 0$, and proceed using backward induction.

Thus, by the induction hypothesis we assume that $V_{t+1}(x_{t+1}, w_{t+1}) = V_{t+1}^2(x_{t+1}) + V_{t+1}^1(w_{t+1})$

We define, $J_t(y_t, z_t, x_t, v_t, w_t) = J_t^1(v_t) + J_t^2(z_t) + J_t^3(y_t) - (S_t + \eta_t)x_t - h_t x_t -$

$\gamma_t w_t$, where

$$J_t^1(v_t) = L_t(v_t) + \gamma_t v_t + \beta E_\xi V_{t+1}^1(v_t - \xi_t) \quad (4.4)$$

$$J_t^2(z_t) = (S_t + \eta_t + h_t - \beta f_t - \alpha_t) z_t \quad (4.5)$$

$$J_t^3(y_t) = (\beta f_t + \alpha_t) y_t + \beta E_{\chi,\omega|\chi_t,\omega_t} E_\xi V_{t+1}^2(y - \xi_t) \quad (4.6)$$

We observe that all of the above functions are convex or linear, thus, we can write

$$\begin{aligned} V_t(x_t, w_t) &= \min_{x_t \le z_t \le y_t; w_t \le v_t \le z_t} J_t(y_t, z_t, x_t, v_t, w_t) = \min_{y_t \ge x_t} \left\{ J_t^3(y_t) + \min_{y_t \ge z_t \ge x_t} \left\{ J_t^2(z_t) \right.\right. \\ &\left.\left. + \min_{z_t \ge v_t \ge w_t} J_t^1(v_t) \right\} \right\} - (S_t + \eta_t) x_t - \gamma_t w_t \end{aligned}$$

Applying Lemma 4.1 on the constrained minimization of $J_t^1(v_t)$ gives us $J_t^1(v_t^{*u}) + H_t^1(w_t) + G_t^1(z_t)$, a constant term, convex function of w_t, and a convex function of z_t, where v_t^{*u} is the unconstrained minimum of function $J_t^1(v_t)$, and $G_t^1(z_t)$=$J_t^1(z_t \wedge v_t^{*u}) - J_t^1(v_t^{*u})$,which refers to the imputed cost to the upper echelon if the manager at upper echelon supplies downstream a lower quantity than what it is optimal from the perspective of the lower echelon. In addition, $H_t^1(w_t) = J_t^1(w_t \vee v_t^{*u}) - J_t^1(v_t^{*u})$, which refers to the cost lower echelon has to incur for carrying excess inventory where $w_t > v_t^{*u}$. Furthermore, we can define $V_t^1(w_t) = H_t^1(w_t) - \gamma_t w_t$ Thus (4.3) can be written as

$$\begin{aligned} V_t(x_t, w_t) &= \min_{y_t \ge x_t} \left\{ \min_{x_t \le z_t \le y_t} \left\{ K_t^2(z_t) \right\} + (\beta f_t + \alpha_t^b) y_t + \beta E_{\chi,\omega|\chi_t,\omega_t} E_\xi V_{t+1}^2(y_t - \xi_t) \right\} \\ &\quad + J_t^1(v_t^{*u}) + V_t^1(w_t) - (S_t + \eta_t) x_t \end{aligned}$$

where,

$$K_t^2(z_t) \quad = \quad J_t^2(z_t) + G_t^1(z_t) \tag{4.7}$$

Furthermore, applying Lemma 4.1 on the constrained minimization of $K_t^2(z_t)$, gives $K_t^2(z_t^{*u}) + H_t^2(x_t) + G_t^2(y_t)$, where z_t^{*u} is the unconstrained minimum of function $K_t^2(z_t)$, and $G_t^2(y_t) {=} K_t^2(y_t \wedge z_t^{*u}) - K_t^2(z_t^{*u})$, which refers to the imputed cost to the manager responsible for forward procurement if the manager of forward procurement makes a decision that allows for less spot procurement than what is optimal from the perspective of the manager responsible for spot procurement. In addition, $H_t^2(x_t) = K_t^2(x_t \vee z_t^{*u}) - K_t^2(z_t^{*u})$, which refers to the excess cost incurred by upper echelon for holding excess echelon inventory at the upper echelon. Also, we define $\hat{H}_t^2(x_t) = H_t^2(x_t) - (S_t + \eta_t)x_t$

Thus (4.3) can be written as,

$$V_t(x_t, w_t) \quad = \quad \min_{x_t \leq y_t} \left\{ K_t^3(y_t) \right\} + J_t^1(v_t^{*u}) + K_t^2(z_t^{*u}) + V_t^1(w_t) + HH_t^2(x_t)$$

where,

$$K_t^3(y_t) \quad = \quad G_t^2(y_t) + J_t^3(y_t)$$

Applying Lemma 4.1 on the constrained minimization of $K_t^3(y_t)$, gives us $K_t^3(y_t^{*u}) + H_t^3(x_t)$, where y_t^{*u} is the unconstrained minimum of function $K_t^3(y_t)$, and, $H_t^3(x_t) = K_t^3(x_t \vee y_t^{*u}) - K_t^3(y_t^{*u})$, which refers to the excess cost incurred by upper echelon for holding excess echelon inventory at the upper echelon.

Equating, $V_t^2(x_t) = \left\{ J_t^1(v_t^{*u}) + K_t^2(z_t^{*u}) + K_t^3(y_t^{*u}) + H_t^3(x_t) + \hat{H}_t^2(x_t) \right\}$, we get

$V_t(x_t, w_t) = V_t^2(x_t) + V_t^1(w_t) \diamondsuit$

Corollary4. 1: The above theorem implies that the top-down base stock policy is optimal. If the decision making process in the above centralized model follows a hierarchy, then the policy that considers the decisions from the upstream manager as inviolable and accommodates the requirements of the downstream manager accordingly is considered to be optimal. The hierarchy in the model is such that the manager who deals with the procurement at the lower echelon is at the bottom of the hierarchy, following the manager who is responsible for the procurement from the spot market, following the manager who procures gasolines from the forward market. This policy follows the traditional framework of base-stock policy such that y_t^{*u} is the base stock level of procurement from the forward market, z_t^{*u} is the base stock level for procurement from the spot market, and v_t^{*u} is the base stock level at the lower echelon. The optimal decision of procurement from the forward market is given by $y_t^* = y_t^{*u} \vee x_t$, similarly, the optimal decision of stocking from the spot market z_t^* is given as $z_t^* = (y_t^{*u} \wedge z_t^{*u}) \vee x_t$, and the allocation to the lower echelon is given by $v_t^* = (z_t^{*u} \wedge v_t^{*u}) \vee w_t$.

It is important to notice that marginal convenience yield, as defined above should be positive, $\delta_t \geq 0$. If the marginal convenience yield were negative then it would create an opportunity for risk free arbitrage. Negative marginal convenience yield entails that a firm can procure a commodity in a spot market at spot price S_t,contract storage from t to $t+1$ paying h, and sell a forward contract at f_t, generating a cash flow with a present value of βf_t, thus, making a positive profit on such transaction. In equilibrium, such phenomenon would not exist as every firm will start short selling commodity, which will increase the spot price and drive marginal convenience yield into positive territory.

Lemma 4.2: Characterization of Optimal Spot Procurement Policies

The base stock procurement level at the lower echelon is larger than or equal to the base stock procurement level from the spot market at the upper echelon, $v_t^{*u} \geq z_t^{*u}$.

Proof: The marginal convenience yield δ_t, is defined as $\delta_t = S_t + h_t - \beta f_t$. From (4.7) we observe that the function $K_t^2(z_t)$ is downward sloping with its slope equal to zero at $z_t^{*u} \geq v_t^{*u}$, where $v_t^{*u}$and z_t^{*u} are the unconstrained minimizer for the functions $J_t^1(v_t)$ and $K_t^2(z_t)$ respectively. The linear term in (4.7) has a positive slope of $\delta_t + \eta_t - \alpha_t$, thus optimal $z_t^{*u} \leq v_t^{*u}$.$\diamondsuit$

The intuition behind the above lemma is that since the manager at the lower echelon is not considering cost to purchase inventory he tends to order a sufficiently large quantity, however, the manager responsible for procurement from the spot market tends to order less because in addition to all the costs taken into account by the lower echelon manger he also needs to consider the cost of acquiring the inventory, and hence the convenience yield and transportation costs, which forces him to order relatively smaller quantity than what is desired at the lower echelon. Thus, the lower echelon manager is willing to offer higher service level than what is optimal from the perspective of the supply chain. Another implication of this Lemma is that whenever we procure from the spot market, any inventory acquired will be immediately shipped to the lower echelon.

4.2.1 Myopic Policy

In this section, we characterize the myopic policies for the two echelon inventory system described in the previous section. We develop conditions according to which the myopic policies obtained are shown to be optimal.

Theorem 4.2: Optimality of Myopic Policies

For stationary demand and constant marginal convenience yield a myopic policy is optimal.

Proof : The function $J_t^1(v_t)$ given in (4.4) can be computed myopically given that the constraint $v_t \geq w_t$ never binds, thus, $H_t^1(w_t)$ equals zero and it is possible to calculate $J_t^1(v_t)$ myopically. This can be shown to be true also for the case of non-decreasing demand. Notice, that the constraint $z_t \geq v_t$ is not a hindrance in obtaining a myopic optimal policy as it does not have any consequence for cost to go, however, the penalty cost for violating this constraint gets folded in the cost of upper echelon. Similarly, analyzing the function $K_t^2(z_t)$ we could infer that it is easy to calculate this function myopically as there are no implications of cost to go. Thus, function $G_t^2(y_t)$ can also be computed myopically, in addition, if we assume that the demand is stationary, and the marginal convenience yield is constant over time periods then the function $K_t^3(y_t)$ can also be computed myopically. The assumption of $y_t^* \geq x_t$ in all time periods leads to $H_t^3(x_t) = 0$, thus the function of cost to go $V_{t+1}^2(x_{t+1})$ is defined in terms of constants and the function $\hat{H}_t^2(x_{t+1})$, which, as shown previously, can be computed myopically for a constant marginal convenience yield, thus the function of cost to go can be computed myopically making it possible to compute the function $K_t^3(y_t)$ myopically. For stationary case $y_t^* \geq y_{t-1}^* - \xi_{t-1}$, ensures that the myopic solution is optimal.$\diamondsuit$

The serial multi-echelon model allows us to obtain an optimal solution and conditions in which myopic policy is optimal. After developing insights from the basic model we extend the model to the multiple retailer case in the following section.

4.3 Multiple Retailers Model

In this Section, we define in detail the procurement and distribution model in Section 4.3.1; Characterization of procurement policies from spot and forward market are developed in Section 4.3.2.

4.3.1 Mathematical Model

This model addresses a two echelon inventory system in which a single commodity is procured at the higher echelon either through spot market or through forward contract transactions, and then this commodity is shipped to the lower echelon as shown in Figure 4.2. Specifically, in our model it means that the gasoline is procured centrally at the terminal through spot and forward market transactions and, thereafter, shipped to the retailers downstream. Since gasoline is reformulated at the terminal, it is not economical to re-sell it in the commodity market; therefore, this system summarizes a make-to-stock manufacturing system, which implies that commodity once bought from the market and reformulated can not be resold in the commodity market. The commodity bought from the forward market arrives with a lead time of one period while spot procurement has a zero lead time. In particular, both spot and forward procurement decisions are made before the demand in the period is realized. Moreover, the transaction cost from the forward procurement α_t is considered to be lower than the spot transaction cost η_t because in spot trade the goods have to be transported with a short notice and this usually results in a higher cost of transportation. In addition, we consider a zero lead time in shipping of gasoline from the terminal to the the retailer i at a transportation cost of γ_t^i. This system operates under periodic review system, where unsatisfied demand at the retailer is completely backlogged. There are N non-homogeneous retailers, which differ on their demand distribution and penalty costs. The cost of each unit of unsatisfied demand is p_t^i in period t for retailer i, and the cost of holding stock at retailer i costs h_t^i per unit in period t, and h_t is per unit holding cost of commodity in period t at the upper echelon. The vector of demand realizations at each retailer in period t, is denoted as $\vec{\xi_t}$ where each realization of demand for retailer i is drawn from the normal distribution given by $\phi_t^i(\xi)$. We denote the aggregate demand of the network as $\zeta_t = \sum_{i=1}^{N} \xi_t^i$

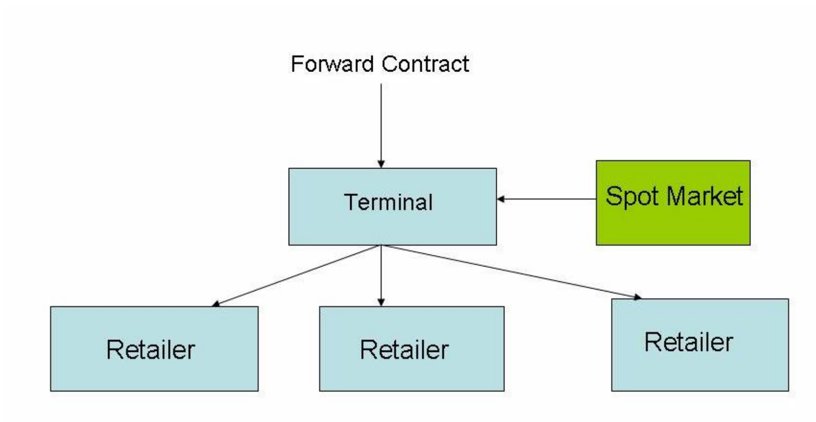

Figure 4.2: Multi-Location Distribution System

The state of the system can be represented as follows, w^i denotes the inventory level at retailer i; $\tilde{\mathbf{w}}$ represents the vector of inventory levels at all the retailers, and the echelon inventory at the warehouse is denoted by x. The allocation of inventory to respective retailers is denoted by v^i where $i = 1, 2..N$, and v^i denotes the inventory position at retailer i. The loss function for each retailer i is given by

$$L_t^i(v^i) = \int_0^{v^i} h_t^i(v^i - \xi_t^i)\phi_t^i(\xi)\, d\xi + \int_{v^i}^{\infty} p_t^i(\xi - v^i)\phi_t(\xi)\, d\xi$$

The overall objective of the firm is to minimize the cost function $V_t^D(x, \tilde{\mathbf{w}})$, which is modeled as a stochastic dynamic program whose state space are represented by short term deviation factor χ_t, long-term equilibrium factor ω_t, the initial echelon inventory at the terminal x, and the vector of initial inventory is at the retailers $\vec{\mathbf{w}}$. We omit χ_t and ω_t from the state space in order to simplify the notation.

$$V_t^D(x, \tilde{\mathbf{w}}) = \min_{\substack{x \leq z \leq y \\ z \geq \sum_{i=1}^N v^i \\ w^i \leq v^i; y \geq 0}} \left\{ \begin{array}{c} (S_t + \eta_t)(z - x) + \beta(f_t + \alpha_t)(y - z) \\ +h_t(z - \sum_{i=1}^N v_t^i) + + \sum_{i=1}^N \gamma_t^i(v^i - w^i) \\ + \sum_{i=1}^N L_t^i(v^i) + \beta E^{\mathbb{Q}} E_{\zeta_t} V_{t+1}^D(y - \zeta_t, \vec{\mathbf{v}} - \vec{\xi_t}) \end{array} \right. \quad (4.8)$$

and,

$$V_{T+1}(x, \tilde{\mathbf{w}}) = (S_{T+1} + \eta_{T+1})(-x)^+ - (S - \eta_{T+1})(x)^+$$

The expectation $E_{\boldsymbol{\zeta}_{\mathbf{t}}}$ consists of N-tuple integral

$$E_{\boldsymbol{\zeta}_{\mathbf{t}}} g(\vec{\xi_t}, \zeta_t) = \int_0^\infty \int_0^\infty \ldots\ldots\ldots \int_0^\infty g(\vec{\xi_t}, \zeta_t) \phi_t^1(\xi_t^1) \phi_t^2(\xi_t^2) \ldots\ldots. \phi_t^N(\xi_t^N) d\xi_t^1 \ldots.. d\xi_t^N$$

and, $ln(S_t) = \chi_t + \omega_t$. For notational simplicity we omit time subscripts from state and decision variables when doing so results in no ambiguity. We denote $E^{\mathbb{P}} \Psi(\chi, \omega) = E_{\chi, \omega | \chi_t, \omega_t} \Psi(\chi, \omega)$, where the expectation is taken under the historic probability measure of the price stochastic process conditional on χ_t and ω_t. The first term in the cost function (4.8) is the cost of procurement from the spot market; second term represents the cost of procurement through forward contract; third term is the holding cost of stock at the upper echelon; the fourth term is the aggregate cost of transportation of the commodity from the terminal to the respective retailers;

the fifth term is the loss function at retailers; the sixth term is the discounted cost to go in the future period. Constraint $y \geq 0$ ensures that backlogs are not carried indefinitely into the future, but are always satisfied in the following period. The constraint $z \geq \sum_{i=1}^{N} v^i$ ensures that the inventory position at upper echelon after spot purchases is higher than the combined inventory position at lower echelon. The constraint $v^i \geq w^i$ implies that only positive amount of inventory is shipped to the retailers, thus, not allowing for any *balancing* of inventory through "lateral" transshipments.

To Simplify the discussion below we introduce the operator $\mathcal{H}_t$. Let f be a function $f : \mathbb{R}^{N+1} \to \mathbb{R}$, then $\mathcal{H}_t[f]$ is defined as follows

$$\begin{aligned}\mathcal{H}_t[f](x, y, z, \tilde{\mathbf{w}}, \tilde{\mathbf{v}}) \quad = \quad & (S_t + \eta_t)(z - x) + \beta(f_t + \alpha_t)(y - z) + h_t(z - \sum_{i=1}^{N} v^i) \quad (4.9)\\ & + \sum_{i=1}^{N} \gamma_t^i(v^i - w^i) + \sum_{i=1}^{N} L_t^i(v^i) + \beta E^{\mathbb{Q}} E_{\zeta_t} f(y - \zeta_t, \vec{\mathbf{v}} - \vec{\xi_{\mathbf{t}}})\end{aligned}$$

Clearly $\mathcal{H}_t[f]$ is a function of $(x, y, z, \tilde{\mathbf{w}}, \tilde{\mathbf{v}})$ as indicated above. However, for notational simplicity we will write it as $\mathcal{H}_t[f]$. Using this notation, we can define V_t^D as

$$V_t^D(x, \tilde{\mathbf{w}}) \quad = \quad \min_{\substack{x \leq z \leq y \\ y \geq 0 \\ w^i \leq v^i,\, i = 1, 2, \ldots N \\ \sum_{i=1}^{N} v^i \leq z}} \quad \mathcal{H}[V_{t+1}^D] \qquad (4.10)$$

Lemma 4.3: Isotonicity of $\mathcal{H}_t$

If $f : \mathbb{R}^{N+1} \to \mathbb{R}$ and $g : \mathbb{R}^{N+1} \to \mathbb{R}$ are such that $f \leq g$ point-wise, then $\mathcal{H}_t[f] \leq$

$\mathcal{H}_t[g]$

Proof: Immediate from $f(y-\zeta_t, \vec{\mathbf{v}}-\vec{\xi_t}) \leq g(y-\zeta_t, \vec{\mathbf{v}}-\vec{\xi_t})$ and from the observation that the probability distributions in the calculation of $E^{\mathbb{Q}}$ and E_ζ are non-negative.

Lemma 4.4: Preservation of Convexity

If $f : \mathbb{R}^{N+1} \to \mathbb{R}$ *is a convex function, then* $\mathcal{H}_t[f]$ *is convex in* $(x, y, z, \vec{\mathbf{v}}, \vec{\mathbf{w}})$

Proof: Since f is a convex function $f(y-\zeta_t, \vec{\mathbf{v}}-\vec{\xi_t})$ is convex in $(y, \vec{\mathbf{v}})$for every demand outcome $\tilde{\xi_{\mathbf{t}}}$, and every price factor outcome $(\chi_{t+1}, \omega_{t+1})$. Since all probabilities are non-negative $E^{\mathbb{Q}}E_{\zeta_t}f(y-\zeta_t, \vec{\mathbf{v}}-\vec{\xi_t})$ is convex in $(y, \vec{\mathbf{v}})$ for every (ω_t, χ_t). The result follows since the rest of the terms in the definition of $\mathcal{H}_t$ are convex.$\diamondsuit$

Lemma 4.5: Convexity of V_t^D :

The function V_t^D *is convex in* $(x, \tilde{\mathbf{w}})$ *for every* (ω_t, χ_t)

Proof: Using backward induction assume that V_{t+1}^D is convex in $(x, \tilde{\mathbf{w}})$. This assumption can easily be verified for V_{T+1}^D. Moreover, if V_{t+1}^D is convex, it follows from Lemma 4.2 that $\mathcal{H}_t[V_{t+1}^D]$ is convex in $(x, y, z, \tilde{\mathbf{w}}, \tilde{\mathbf{v}})$ for every (ω_t, χ_t), and since the minimization in (4.10) is taken over a convex set, it follows that V_t is convex in $(x, \tilde{\mathbf{w}})\diamondsuit$.

4.3.2 Characterization of Procurement Policies

This subsection develops insights on procurement policies from spot and forward markets. In this regards, we obtain a set of first order conditions where equations (4.11), (4.12) and (4.13) represents first order condition for decision variables, z,y, and v^i respectively,

$$\begin{aligned}
S_t + h - \beta f_t + \eta_t - \alpha_t - \lambda - \mu + \pi &= 0 \quad (4.11)\\
\beta f_t + \beta\alpha_t + \beta D_y^+[E^{\mathbb{Q}} E_{\zeta_t} V_{t+1}^D(y - \zeta_t, \vec{\mathbf{v}} - \vec{\xi_t})] - \pi &= 0 \quad (4.12)\\
\lambda + \gamma^i + D_{v^i}^+[L_t(v^i)] - k^i + \beta D_{v^i}^+[E^{\mathbb{Q}} E_{\zeta_t} V_{t+1}^D(y - \zeta_t, \vec{\mathbf{v}} - \vec{\xi_t})] &= 0 \quad (4.13)\\
\mu(z - x) &= 0 \quad (4.14)\\
\lambda(z - \sum_{i=1}^{N} v^i) &= 0 \quad (4.15)\\
\pi(y - z) &= 0 \quad (4.16)\\
k^i(v^i - w^i) &= 0 \quad (4.17)
\end{aligned}$$

where, μ, λ, π, and k^i are Lagrangian multipliers associated with constraints $z \geq x$, $z \geq r$, $y \geq z$, and $v^i \geq w^i$ respectively. The optimal solution should satisfy the above set of equations, and as a result, we can observe the following properties on optimal procurement policies. Let $z^*$and y^* denote the optimal echelon inventory after procurement from spot and forward market respectively.

Property 1: *if* $y^* > z^* > x$ *then* $\pi = 0$ *,* $\mu = 0$ *and* $\lambda = S_t + h_t - \beta f_t + \eta_t - \alpha_t$.

This property follows from (4.11), (4.14), and (4.16), and it implies that when it is optimal to procure from both spot and forward markets, the value of the Lagrange multiplier λ associated with the allocation constraint (4.15) is given by the convenience yield, implied by the commodity markets, adjusted by the transaction cost differential $\eta_t - \alpha_t$. We will denote this adjusted convenience yield as $\hat{\delta}_t = \delta_t + \eta_t - \alpha_t$.

Property 2: *if* $y^* > z^* = x$ *then* $\pi = 0$*,* $\mu \geq 0$*, and* $\lambda \leq \hat{\delta}_t$

This property states that if there is a positive procurement from the forward market but it is not optimal to procure from the spot market then (4.14) implies that $\mu \geq 0$, and (4.16) implies $\pi = 0$. Hence it follows from (4.11) that $\lambda \leq \hat{\delta}_t$.

Property 3: *If $y^* > z^*$,then λ and μ cannot both be equal to zero*

Since, $\hat{\delta}_t > 0$, and $y^* > z^*$ implies $\pi = 0$, it follows from (4.11) that $\lambda + \mu > 0$. We can examine two cases emanating from this property: a) If $\mu = 0$, which occurs when there is a spot market procurement, then it must also satisfy $\lambda > 0$. In this case (4.15) implies that $z = \sum_{i=1}^{N} v^i$, showing that whenever we procure from the spot market, all stocks at the upper echelon must be fully allocated to the downstream retailers; there is no physical inventory at the upper echelon. (b) if $\lambda = 0$, which will happen when there is physical inventory at the upper echelon as implied by (4.15), we must have $\mu > 0$. In this case (4.14) implies it is not optimal to procure from the spot market $z^* = x$. This is the converse of the implication of case (a), namely, if an optimal policy does not allocate the echelon inventory fully to the downstream retailers, then it must not procure from the spot market.

Since the transaction cost adjusted convenience yield is positive, $\hat{\delta}_t > 0$ we can expect under common operating conditions some degree of forward procurement, $y^* > z^*$. However, it is also conceivable that an optimal policy prescribes no forward procurement, $y^* = z^*$; this may happen, for example, in anticipation of a downturn in demand for subsequent periods. Properties 4 and 5 are related to this particular case.

Property 4: *If $y^* = z^* > x$ then $\mu = 0, \pi \geq 0,$ and $\lambda > \hat{\delta}_t$*

This property highlights that if an optimal policy suggests no procurement from the forward market but there is a procurement from the spot market then (4.16) implies $\pi \geq 0$, and (4.14) implies $\mu = 0$. Hence it follows from (4.11) that $\lambda > \hat{\delta}_t$.

Property 5: *if $y^* = z^* = x$ then $\lambda \geq 0$, $\pi \geq 0$, and $\mu \geq 0$*

In this case, when there is no procurement of either kind then nothing specific can be said about the value of λ

In the literature on one-warehouse multi-retailer, researchers have dealt with the allocation constraint by dualizing it, and devising methods to obtain the value of the dual variable through the internal characteristics of the network. However, in our model, we have the flexibility of instantaneous procurement from the spot market; therefore, the commodity market equilibrium influences the value of the dual variable of the allocation constraint,λ, or it provides bounds as described in the properties above. Notice that from this model the value of the dual variable can be determined without making a *balancing assumption*. One of the contributions of our model is to propose a method to obtain the cost of relaxing the allocation constraint through market equilibrium in conjunction with the internal parameters of network instead of exclusively relying on internal parameters of the network as has been the solution methodology in the literature. This shall enable us to obtain better procurement and distribution policies for traded commodities, which shall allow us to manage gasoline distribution operation more efficiently.

4.4 Lagrangian Relaxation Approach

As the number of retailers increases in the network the state space of the problem increases, and due to the *curse of dimensionality* it is very difficult to obtain an optimal solution to this problem, therefore, we proceed to develop bounds on the cost function. In this regard, we follow the Lagrangian relaxation approach. In section 4.4.1 we formulate the Lagrangian relaxation model and show that it can be decomposed into a sum of $N + 1$ simpler dynamic programs. In section 4.4.2 we develop a lower bound for the Lagrangian relaxation model and in section 4.4.3 we discuss a limited look-ahead approach to obtain approximate solutions to this model. In section 4.4.4 we discuss the heuristics to estimate the value of Lagrangian multiplier.

4.4.1 Model Formulation

The mathematical model described below belongs to the family of *weakly coupled dynamic programs*, where several dynamic programs are linked together through a common constraint. In our model the *allocation constraint* couples individual dynamic programs associated with the decisions of each retailer with the overall dynamic program of the system. We will attempt to decouple it by dualizing the allocation constraint so that conditional on the value of the multiplier it is feasible to have a solution for the problem by solving the individual dynamic programs in order to achieve bounds on the cost function of the original problem. For a more complete discussion on weakly coupled dynamic programs we refer readers to Adelman and Mersereau (2004) and Caro and Gallien (2007). We next proceed to relax the constraint $z_t \geq \sum_{i=1}^{N} v_t^i$ for all the periods. Here we make an assumption that the value of Lagrange multiplier will be fixed at λ for all subsequent periods. Thus, the objective function can be written as

$$V_t^{\lambda}(x_t, \tilde{\mathbf{w}}_{\mathbf{t}}) = \min_{\substack{x \leq z \leq y \\ y \geq 0 \\ w^i \leq v^i}} \mathcal{H}_t^{\lambda}[V_{t+1}^{\lambda}] \tag{4.18}$$

where,

$$V_{T+1}^{\lambda}(x_{T+1}, \mathbf{w}_{\tilde{\mathbf{T}}+\mathbf{1}}) = (S_{T+1} + \eta_{T+1})(-x_{T+1})^+ - (S_{T+1} - \eta_{T+1})(x_{T+1})^+$$

and the operator $\mathcal{H}_t^\lambda$ is defined as

$$\mathcal{H}_t^\lambda[V] \quad = \quad \mathcal{H}_t[V] - \lambda(z - \sum_{i=1}^{N} v^i)$$

for any scalar $\lambda \geq 0$. It is clear that $\mathcal{H}_t^\lambda$ is also isotonic and preserves convexity as established in Lemma 4.1 and Lemma 4.2 for $\mathcal{H}_t$.

Theorem 4.2: Lower Bound on V_t^D

The Lagrangian relaxation V_t^λ is a lower bound on V_t^D for any non-negative value of the parameter λ

Proof: We proceed by induction by assuming $V_{t+1}^\lambda \leq V_{t+1}^D$. It can be verified that this assumption is true in period $T+1$. Then we can observe that

$$\begin{aligned} V_t^\lambda = \min_{\substack{x \leq z \leq y \\ y \geq 0 \\ w^i \leq v^i}} \mathcal{H}_t^\lambda[V_{t+1}^\lambda] \leq \min_{\substack{x \leq z \leq y \\ y \geq 0 \\ w^i \leq v^i}} \mathcal{H}_t^\lambda[V_{t+1}^D] \leq & \\ \leq \min_{\substack{x \leq z \leq y \\ y \geq 0 \\ w^i \leq v^i \\ z \geq \sum_{i=1}^N v^i}} \mathcal{H}_t[V_{t+1}^D] \quad = \quad V_t^D & \end{aligned} \tag{4.19}$$

The first inequality follows since $\mathcal{H}_t^\lambda[V_{t+1}^\lambda] \leq \mathcal{H}_t^\lambda[V_{t+1}^D]$ point-wise as $\mathcal{H}_t^\lambda$ is isotonic (Lemma 3). The second inequality follows from the definition of $\mathcal{H}_t^\lambda$ as the program in the left is a relaxation of the program in the right and V_{t+1}^D is independent of λ.

Theorem 4.3: Decoupling of V_t^λ

$V_t^\lambda(x, \tilde{\mathbf{w}})$ is convex in $(x, \tilde{\mathbf{w}})$ and can be decomposed as, $V_t^\lambda(x, \tilde{\mathbf{w}}) = h_t(x; \lambda) + \sum_{i=1}^N g_t^i(w^i; \lambda)$, where h_t is convex in x and g_t^i is convex in w^i for all i and for any value of λ

Proof: The validity of the theorem for time $T+1$ is immediate from (4.18) by setting $h_{T+1}(x; \lambda) = (S_{T+1} + \eta_{T+1})(-x)^+ - (S_{T+1} - \eta_{T+1})(x)^+$, and $g_{T+1}^i(w; \lambda) = 0$, for all i . We proceed inductively by assuming that Theorem 4.3 holds for time $t+1$, thus, $V_{t+1}^\lambda(x, \tilde{\mathbf{w}}) = h_{t+1}(x; \lambda) + \sum_{i=1}^N g_{t+1}^i(w^i; \lambda)$. Hence, (4.18) can be defined as

$$\mathcal{H}_t^\lambda[V_{t+1}^\lambda] = J_t^1(z; \lambda) + J_t^2(y; \lambda) + \sum_{i=1}^N f_t^i(v_t^i; \lambda) - (S_t + \eta_t)x_t - \sum_{i=1}^N \gamma_t^i w_t^i \quad (4.20)$$

$$J_t^1(z; \lambda) = (S_t + h_t - \beta f_t + \eta_t - \alpha_t - \lambda)z_t \quad (4.21)$$

$$J_t^2(y; \lambda) = \beta(f_t + \alpha_t)y_t + \beta E^{\mathbb{Q}} E_{\boldsymbol{\zeta_t}} h_{t+1}(y_t - \zeta_t; \lambda) \quad (4.22)$$

$$f_t^i(v_t^i; \lambda) = L_t^i(v_t^i) + (\gamma_t^i + \lambda - h_t)v_t^i + \beta E_{\boldsymbol{\zeta_t}} g_{t+1}^i(v_t^i - \xi_t^i; \lambda) \quad (4.23)$$

It follows from the induction hypothesis that $J_t^2(y; \lambda)$is convex in y_t , $J_t^1(z : \lambda)$ is convex in z, and the functions f_t^i are convex in v^i for all values of λ. Hence, $\mathcal{H}_t^\lambda[V_{t+1}^\lambda]$ is convex in $(x, y, z, \tilde{\mathbf{v}}, \tilde{\mathbf{w}})$. Now we can write equation (4.18) as

$$V_t^\lambda(x, \tilde{\mathbf{w}}) = \min_{z \geq x} \left\{ J_t^1(z; \lambda) + \min_{\substack{y \geq z \\ y \geq 0}} J_t^2(y; \lambda) \right\} + \sum_{i=1}^N \min_{v^i \geq w^i} f_t^i(v^i; \lambda) - (S_t + \eta_t)x - \sum_{i=1}^N \gamma_t^i w^i$$

It follows from Lemma 4.1 that $J_t^2(y;\lambda) = J_t^2(y^*;\lambda) + H_t^2(z;\lambda)$

$$H_t^2(z;\lambda) \quad = \quad J_t^2(y \vee z;\lambda) - J_t^2(y^u;\lambda) \tag{4.24}$$

where, y^u is the unconstrained minimizer of $J_t^2(y;\lambda)$. The function $H_t^2(z;\lambda)$ is convex and non decreasing. Now we can write equation (4.18) as

$$V_t^{\lambda}(x,\tilde{\mathbf{w}}) \quad = \quad \min_{z \geq x} K_t^1(z;\lambda) + \sum_{i=1}^{N} \min_{v^i \geq w^i} f_t^i(v^i;\lambda) - (S_t + \eta_t)x - \sum_{i=1}^{N} \gamma_t^i w^i$$

where,

$$K_t^1(z;\lambda) \quad = \quad J_t^1(z;\lambda) + H_t^2(z;\lambda) \tag{4.25}$$

It follows from Lemma 4.1 that $\underset{z \geq x}{\min} \; K_t^1(z;\lambda) = K_t^1(z^u;\lambda) + H_t^1(x;\lambda)$, where

$$H_t^1(x;\lambda) \quad = \quad K_t^1(z \vee x;\lambda) - K_t^1(z^u;\lambda) \tag{4.26}$$

where z^u is the unconstrained minimizer of function $K_t^1(z;\lambda)$; the function $H_t^1(z;\lambda)$ is convex and nondecreasing. We define

$$h_t(x;\lambda) \quad = \quad K_t^1(z^*;\lambda) + H_t^1(x;\lambda) - (S_t + \eta_t)x_t \tag{4.27}$$

$$g_t^i(w^i;\lambda) \quad = \quad \min_{v^i \geq w^i} f_t^i(v^i;\lambda) - \gamma_t^i w_t^i \tag{4.28}$$

Thus, $h_t(x;\lambda)$ is convex in x, and g_t^i is convex in w^i, and we establish that $V_t^{\lambda}(x,\tilde{\mathbf{w}}) = h_t(x;\lambda) + \sum_{i=1}^{N} g_t^i(w^i;\lambda)$, and V_t^{λ} is convex in $(x,\tilde{\mathbf{w}})\diamondsuit$.

Theorem 4.3 elucidates that it is possible to decouple the larger dynamic program for the entire system into various smaller dynamic programs one for each terminal and one for the upper echelon. As a result, the complexity of the problem increases linearly with the number of downstream retailers N. These sub programs are represented as a function of the Lagrange multiplier λ applied in the relaxation of the allocation constraint.

Lemma 4.6: Characterization of procurement policy as implied by $\mathbf{V_t^\lambda}$

Procurement policy from spot and forward markets are base stock policies for a given value of λ, and are given as following

a) The optimal procurement policy in forward market is order up to y^u

b) The optimal spot market procurement policy is order up to z^u

Proof: a) From (4.25) we get the convexity of $K_t^1(z;\lambda)$ in z. z^u is the unconstrained minimum of $K_t^1(z;\lambda)$. Thus, spot procurement is made up to z^u if the inventory level x is less than z^u.

Proof: b) From (4.22) we get the convexity of $J_t^2(y;\lambda)$ in y. y^u is the unconstrained minimum of $J_t^2(y;\lambda)$. Thus, spot procurement is made up to y^u if the spot procurement level z^u is less than $y^u\diamondsuit$.

Next, we characterize the distributive policy to the retailers. In general, retailers downstream tends to order more from upper echelon than what upper echelon would desire to have as a service level at downstream retailers. This primarily occurs because retailers do not incur convenience yield and transportation costs of procurement upstream. Hence, lower costs accounted for holding inventory leads to higher service level. Therefore, in order to align the procurement order from downstream up to the level deemed optimal by upper echelon, upper echelon charges λ for each unit procured by downstream.

Lemma 4.7: Characterization of distribution policy as implied by $\mathbf{V_t^\lambda}$

The optimal procurement at the retailer v^{i*} is non increasing in λ.

Proof: From (4.23) $\frac{\partial^2 f_t^i(v^i;\lambda)}{\partial v^i \partial \lambda} \geq 0$.Hence, optimal v_t^{i*} is non-increasing in $\lambda\diamondsuit$.

The upper echelon uses λ as a lever to adjust the stocking levels at the lower echelon. The value of λ is ascertained through the sequence of properties developed in Section 4.3.2, and its impact on the lower echelon is given by Lemma 4.7.

Lemma 4.8: Characteristics of Optimal Procurement Policy

There exists an optimal solution to procurement and distribution policy which is computationally feasible if in every period $y_t^* \geq z_t^* \geq x_t$, and all the elements in vector $\{\hat{\delta}_t, \hat{\delta}_{t+1}, \hat{\delta}_{t+2}..........\hat{\delta}_N\}$ are known at time t.

Proof: Following the arguments in Property 1, if $y_t^* \geq z_t^* \geq x_t$ is satisfied in each period then $\lambda = \hat{\delta}_t$. Hence, if the values of $\hat{\delta}$ are known for subsequent periods then all future values for λ will also be known resulting in an optimal solution. $\diamondsuit$

We know from Property 1 that for optimal solution to exists there has to be procurement in the spot market and the forward market in every period. This optimal solution is obtained assuming that the marginal convenience yield and transportation costs either remains same or are known for each period. Moreover, the value of the Lagrange multiplier is entirely determined by market equilibrium in each period. To our knowledge, this is the first model that reports an optimal solution under more general framework. In the literature, it has been shown that if the balancing condition is not violated then it is possible to attain an optimal solution. However, our result illustrates that even without making a balancing assumption it is possible to obtain the optimal solution, given that marginal convenience yield and transportation costs are known for future periods. Under more general conditions it may not be possible to ascertain the value of marginal convenience yield for future periods as the prices are evolving in continuous time. Thus, it will not be possible to calculate optimal value of r_t^* and λ for general cases. Hence, we discuss some of the approximations for calculating the values of r_t^* and λ in Section 4.4.4

4.4.2 Lower Bound

For a given value of λ it is possible to calculate the function f_t^i for each retailer, similarly for a realized state space of χ and ω it is possible to compute the function J_t^1. However, it is not possible to calculate the function J_t^2 because it is not trivial to compute the transitional probability from state (χ_t, ω_t) to $(\chi_{t+1}, \omega_{t+1})$. Therefore, we propose a method to approximate these transitional probabilities through simulation. We propose to chose a confidence interval for the value of χ and ω, which will be time dependent and it is expected to increase with time. The state space variable χ follows a log-normal distribution, which has a mean of 0 and variance given by $(1-e^{-2kt})\sigma_\chi^2/2k$. Thus, the variance of χ increases with time, but it is upper bounded by $\sigma_\chi^2/2k$. On the other hand, the state space variable ω follows a normal distribution with mean μt and variance $\sigma_\omega^2 t$. Hence, mean and variance of ω grows linearly in time. We will now discretized the state space of χ and ω, which will grow with time, and simulate the probability transition from state space at time t to state space in time $t+1$. The confidence interval for ω and χ is given by $\mu t \pm z\sigma_\omega^{'} t$ and $0 \pm z^{'}(1-e^{-2kt})\sigma_\chi^2/2k$ respectively, where $z^{'}$ represents the number of standard deviations from the mean. The accuracy in calculation of lower bound will improve with more fine partition of the state space, however, with more finer partitions the complexity in numerical calculation increases. Hence, right balance has to be maintained to obtain a bound that is feasible to calculate and is also accurate.

We propose a grid of state spaces with width of 0.01 for state space of both χ

and ω. The center point of a square in a grid represents the coordinate of the state space. The state space transitions to a different state space next period, however, since χ and ω are continuous variables the transition point to a new grid will be a continuum on the grid space. Any transition point in the square of a grid will be approximated to have the coordinates of the center of the square as the state space in order to discretized the continuous state space. The purpose of discretizing the state space over a confidence interval is to obtain computational feasibility, but there are additional challenges in dealing with it. Such as, how to deal with the following cases; if the transition in next period occurs to a state space that is outside the considered confidence interval or if the transition occurs from a state that is outside the limits of confidence interval into a state that is in the limits of confidence interval. For computational feasibility we ignore such cases. In the calculation of V_t^{λ} we need to calculate the dynamic program that has three dimension state space; we have already established the method to calculate the transition probabilities for state variables χ and ω; in addition, it is simple to calculate the transition probability for demand. Also notice, that the evolution of demand is independent of the state variables χ and ω.

In the calculation of the dynamic program, function f_t^i is calculated for each realization of demand and it is independent of the evolution of state spaces χ and ω; function J_t^1 can be calculated myopically for each realization of state space χ and ω, and this function is independent of the realization of demand; function J_t^2 is the one that is computed dynamically and has to be evaluated for each realization of ξ, χ and ω. Based on our choice of $z^{'}$ we determine the percentage of transitions that occur to the state spaces outside the confidence interval. The objective is to choose $z^{'}$ such that the percentage of transitions to the state space outside the confidence interval is reduced without significantly increasing the width of the confidence interval as larger confidence interval leads to more computational challenges by marginally improving the accuracy of the calculations. This described calculation procedure allows us to compute the lower bound. Any V_t^λ is a lower bound, but we will obtain the best lower bound by searching a value of λ that maximizes the value of V_t^λ

4.4.3 Upper Bound

An upper bound on $h_t(x_t;\lambda)$ can be computed using the concept of *limited look-ahead policies*, according to which the dynamic program is truncated after the finite number of steps, and at each stage a decision is based on look-ahead of a limited number of stages. The higher the number of stages the better is the approximation, but more complex is the computation. We use two-stage look-ahead policy to compute the decision variable at time t. This, assumes that planning horizon ends after two periods. We define,

$$W_t(y_t, x_t) = \beta(f_t + \alpha_t^b)y_t + (\hat{\delta}_t - \lambda)^+ x_t - (\lambda - \hat{\delta}_t)^+ y_t - (S_t + \eta_t^b)x_t \quad (4.29)$$

The optimal solution of the given dynamic program is used to calculate the upper bound on the cost function.

$$\tilde{h}_t(x_t;\lambda) = \min_{y_t} \left\{ W_t(y_t, x_t) + E^{\mathbb{Q}} E_{\varsigma t+1} \min_{y_{t+1}} \left\{ W_{t+1}(y_{t+1}, x_{t+1}) + \tilde{h}_{t+2}(y_{t+1} - \zeta_{t+1};\lambda) \right\} \right\}$$

s.t $y_t \geq x_t$;

where $\tilde{h}_t$ is the approximation such that the dynamic program terminates at time $t+2$. The optimal solution obtained from $\tilde{h}_t$ can be used to calculate the upper bound. Also, the prices for the future periods are calculated as the expectation conditional on the price parameters observed at time t. For the computational purposes, lets understand how the approximation of optimal policy y_t can be calculated at time t. We know that since the dynamic program terminates at time $t+2$, the left over inventory is salvaged and the back-orders are fulfilled. Therefore,

$$\tilde{h}'_{t+2}(x_{t+2}) = \begin{cases} -S_{t+2} - \eta^b_{t+2}, & x_{t+2} < 0 \\ -S_{t+2} + \eta^b_{t+2}, & x_{t+2} > 0 \end{cases}$$

Thus, $E^{\mathbb{Q}} E \tilde{h}'_{t+2}(y_{t+1} - \zeta_{t+1}) = -E^{\mathbb{Q}}(S_{t+2}) - \eta^b_{t+2} + 2\eta^b_{t+2}\Phi_{t+1}(y_{t+1})$

From (4.29) $\partial R_{t+1}/\partial y_{t+1}) = \beta \hat{f}_{t+1} + \beta\alpha^b_{t+1} - (\lambda - \hat{\delta}_{t+1})^+ - \beta\hat{f}_{t+1} - \beta\eta^b_{t+2} + 2\eta^b_{t+2}\Phi_{t+1}(y_{t+1})$,this leads to

$$\Phi_{t+1}(y_{t+1}) = \frac{(\lambda - \hat{\delta}_{t+1})^+ + \beta\eta^b_{t+2} - \beta\alpha^b_{t+1}}{2\eta^b_{t+2}} \tag{4.30}$$

Here, it is important to notice that marginal convenience yield $\hat{\delta}_{t+1}$ is approximated by observing the price trajectory at time t. The above expression will yield different stocking policies conditional on the current price parameters χ_t and ω_t. It is important to notice, that this stocking policy can be calculated as a closed form solution for each sample path without having to calculate the dynamic program, hence it is simple to implement. Similarly,

$$h'_{t+1}(x_{t+1}) = \begin{cases} (\hat{\delta}_{t+1} - \lambda)^+ - S_{t+1} - \eta^b_{t+1}, & y_{t+1} > y_t - \xi_t \\ h - \lambda + \beta E h'_{t+2}(x_{t+1}), & y_{t+1} < y_t - \xi_t \end{cases}$$

We know that, $\beta E h'_{t+2}(x_{t+2}) = -\beta f_{t+1} - \beta\eta^b_{t+2} + 2\beta\eta^b_{t+2}\Phi_{t+1}(x_{t+1})$; Furthermore, $h-\lambda+\beta Eh'_{t+2}(x_{t+1}) = \delta_{t+1}-\lambda-S_{t+1}-\beta\eta^b_{t+2}+2\beta\eta^b_{t+2}\Phi_{t+1}(x_{t+1})$;Thus,

$$\begin{aligned} E^{\mathbb{Q}}E_\xi h_{t+1}(y_t-\xi_t) &= -E^{\mathbb{Q}}(S_{t+1}) + \left\{(\hat{\delta}_{t+1}-\lambda)^+ - \eta^b_{t+1}\right\} \\ &\quad -\left\{(\hat{\delta}_{t+1}-\lambda)^+ - \delta_{t+1}+\lambda\right\}(\Phi_t(y_t-y_{t+1})) + 2\beta\eta^b_{t+2}R(y_t) \end{aligned}$$

where, $\beta\eta^b_{t+2} = \eta^b_{t+1}$, and, $R(y_t)$ is the convolution of demand over two periods such that

$$R(y_t) = \int_0^{y_t-y_{t+1}} \Phi_{t+1}(y_t-\xi_t)\phi_t(\xi_t)d\xi_t$$

and,

$$\begin{aligned} D_{y_t}J_t(y_t,x_t) &= \alpha^b_t - (\lambda-\hat{\delta}_t)^+ - \left\{(\hat{\delta}_{t+1}-\lambda)^+ - \delta_{t+1}+\lambda\right\}\left\{\Phi(y_t-y_{t+1})\right\} \\ &\quad +2\beta\eta^b_{t+2}R(y_t) + \left\{(\hat{\delta}_{t+1}-\lambda)^+ - \eta^b_{t+1}\right\} \end{aligned}$$

Equating the above derivative to zero will yield y_t, which will also depend upon y_{t+1} that can be calculated from (4.30). Hence, the upper bound $\bar{h}_t(x_t;\lambda)$ can be obtained by calculating the procurement cost along the sample path based on the optimal policy obtained from the limited look ahead policy.

4.4.4 Approximation of Lagrangian Multiplier

In the Section 4.4.4 we characterized the optimal procurement policies and developed an understanding on the value of λ based on the optimal policies. However, in this section we develop a heuristic to approximate the value of Lagrangian multiplier. This heuristic uses the properties developed in Section 4.3.2. We define $r_t = \sum v^i_t$

Theorem 4.4: Heuristic for Lagrange Multiplier

Lagrange multiplier has the following characteristics iff $y_t^* \geq z_t^*$.

a) if r_t calculated at $\lambda = \hat{\delta}_t$ is such that $r_t \geq x_t$, then $\lambda^* = \hat{\delta}_t = \delta_t + \eta_t^b - \alpha_t^b$

b) if r_t calculated at $\lambda = 0$ is such that $x_t \geq r_t$, then $\lambda^* = 0$

c) else, approximate λ^*such that $\lambda^* = [\lambda : r_t = x_t]$

d) *If, for a given λ, $y_t^* < z_t^*$ then approximate λ^*such that $\lambda^* = [\lambda : y_t^* = z_t^*]$*

Proof: a) if $r_t \geq x_t$ then $z_t = r_t \geq x_t$, thus from Property 1 $\lambda^* = \hat{\delta}_t = \delta_t + \eta_t^b - \alpha_t^b$

b) if $x_t \geq r_t$ then $z_t = x_t \geq r_t$, thus $\lambda^* = 0$

c) It is an approximation such that λ takes the value for which r_t becomes equal to x_t.

d) since z_t^* is decreasing in λ, the approximate value of λ should at least be such that $y_t^* = z_t^*$,$\diamondsuit$ where y_t^* is determined as given by limited look-ahead policy in Section 4.4.3

We developed structural properties on the Lagrange multiplier that are instrumental in developing the approximation to calculate the value of the Lagrangian multiplier. The value of the Lagrangian multiplier denotes the imputed cost from the retailer to the upper echelon. If the imputed cost is high then there will be procurement from spot and forward market. If there is a positive procurement from spot and forward market then the value of the Lagrange multiplier can be determined from the market equilibrium through the spread in the spot and futures price, and the transportation cost differential. If there is no procurement from the spot market then the value of Lagrangian multiplier is less than $\hat{\delta}_t$, where its value is used as a lever to ensure that there is no procurement from the spot market. It is also possible that the value of Lagrange multiplier may exceed $\hat{\delta}_t$ if $y_t^* < z_t^*$ as the procurement manager from upper echelon may not deem it optimal to procure from the forward market in wake of a possible demand downturn in future periods, however, a retailer downstream experiencing stock out and substantially high penalty cost will drive the value of the Lagrange multiplier higher.

4.5 Computational Results

In this section we illustrate through simulation the performance of the approximate policies developed in the previous sections. We simulate 50 week planning horizon with 1000 price paths and 50 demand paths, which leads to simulating 50,000 planning horizons. The purpose of this numerical study is two-folds; first to ascertain the conditions under which the performance of bounds on the cost function, developed using market equilibrium information, is tight; second to understand better the dynamics of the model such as the impact of demand and price variability, penalty costs, and transportation costs on procurement policies from spot and forward market. This research is motivated by the distribution of gasoline in the supply chain of Chevron, however, for the purpose of gaining insight into the model we will simulate our model based on the data for crude oil. We use the parameters obtained by Schwartz and Smith (2000) for crude oil data to simulate our model. These parameters and initial conditions of the stochastic process are shown in Table 3. 2.

4.5.1 Stationary Demand and Identical Retailers

Table 4.1 elucidates that bounds developed using the proposed approximate policy yields tight bounds under a range of stochastic process parameters and operating parameters. *Effect of penalty cost:* With increase in penalty cost we observe that the proportional procurement from spot market increases, this occurs because retailers downstream are willing to trade additional procurement costs from the spot market with the cost of backlogging demand. This effect leads to the reduction of inventory at the central warehouse. In addition, we observe that the bounds on the cost function are tighter at high penalty cost because higher penalty cost for retailers downstream compels the terminal manager to procure frequently from the spot market, this allows the firm to determine the marginal cost of a unit transferred to the downstream retailers through market equilibrium. Consequently, unravelling of true cost of unit transferred in each period leads to tighter bounds. *Effect of coefficient of variation* Increase in the coefficient of variation leads to higher proportion of procurement from the forward market , this also leads to higher inventory at the terminal because higher uncertainty in demand requires the manager of terminal to carry higher level of stocks in order to be able to respond to the imbalance of inventory at the retailers caused due to higher variations in demand. Since the proportional procurement from spot market reduces it makes it relatively harder to ascertain true cost of units transferred to downstream, which leads to wider gap in the bounds.

Effect of short term fluctuations : High volatility in short term fluctuations leads to higher futures prices, which on average translates to lower marginal convenience yield. We do not witness any significant effect on the procurement pattern due to short-term fluctuations in price in the presence of high penalty costs because these fluctuations does not effect the optimal stocking policy of a retailer at high penalty cost. However, if the penalty cost is low the optimal stocking levels of retailers are more sensitive to the higher fluctuations in price, requiring higher proportion of procurement from the spot market due to decrease in marginal convenience yield. We do not observe any change in inventory at the warehouse due to short term fluctuations in price. However, the total inventory in the system reduces because of non-linearity in optimal stocking levels, which decreases more sharply to higher spot prices than the relative increase in stocking level due to lower spot prices. Similar effects are observed on increase in fluctuations of long-term deviations in price, decrease in the mean reverting coefficient. *Effect of drift :* Increase in the magnitude of negative drift factor leads to higher backwardation in prices that translates to higher marginal convenience yield. Intuitively, as the cost of marginal convenience yield increases the inventory levels in the system should decrease,however, in contrast to our intuition we observe that the inventory levels increase. This result occurs because as the cost of marginal convenience yield increases it increases the cost of operating in the spot market, as a result, higher proportion of inventory is procured from the forward markets. In addition, higher level of inventory is desired in the system to mitigate the risk of higher demand because of the reduced flexibility to procure from the spot market as it is more costly to operate in the spot market . Moreover, the percentage of inventory held at downstream retailers increases since the higher cost of marginal convenience yield ensures stocking inventory at a location that is closer to consumers in order to have more flexibility to respond to demand variations.

		Volatility of Short Term Factor	Volatility of Long Term Factor	Drift	Mean Reversion	Correlation coefficient	Coefficient of Variation	Penalty Cost
		σ_χ	σ_ω	μ	κ	ρ	cv	p
	Base value	0.5	0.35	-0.3	0.2	0.8	0.2	10
Spot trade	49.62	53.64	55.12	35.8	58.2	50.97	31.68	62.3
Forward trade	48.392	44.326	42.8	62.2	39.66	47.0	66.34	35.7
Orders from rack	96.3	96.19	96.3	96.22	96.3	96.28	96.33	96.33
Warehouse inventory	6.95	6.956	6.48	9.6	5.25	6.8	14.87	5.2
System inventory	18.64	10.56	12.1	19.8	17.98	16.42	41.1	13.5
Period-end inventory	0.3457	0.194	0.21	0.4	0.29	0.3	0.78	0.35
Upper Bound Cost	93,943	97,173	95,939	82,330	106,220	94,272	93,874	94,673
Lower Bound Cost	93,291	94,073	95,203	81,676	105,690	93,607	92,724	94,107

Table 4.1: Sensitivity Analysis of Procurement Model

4.5.2 Non Stationary Demand and Non-Identical Retailers

After establishing the robustness of our model for the stationary case we turn our attention to the scenario where retailers are non-identical and demand is non-stationary, such cases have been traditionally known as being difficult to solve optimally. Table 4.2 depicts the performance of bounds when demand is non-stationary considering that there are five non-identical retailers downstream with the following vector of mean demand [10,15,20,25,30], and each has an identical coefficient of variation. In the literature of One Warehouse Multi Retailer optimal policies are obtained by making a balancing assumption. These policies are a good approximation in a case of stationary demand, however, these policies perform poorly under the case of Non-stationary demand. Significantly, in this research we derive approximate policies that perform very well also under the case of Non-Stationary demand. As illustrated in Table 4.2 we allow for demand to be cyclic with degrees of frequency varying from 1 to 12, where frequency refers to number of demand cycles in a year. The coefficient of variance is kept constant by allowing for standard deviation to also follow the same cycle as the mean of demand. Demand for retailer i in period t is given by the following equation

$$\xi_t^i = \mu^i + 5 * sin[C * (t-1) * \pi/(24.5)]$$

where, μ^i is the mean of the demand for retailer i, and C is the frequency of demand cycles. Amplitude of the cyclic demand is 5 for all the retailers. Interestingly, even for a case when we observe one demand cycle each month, the bounds developed using the proposed methodology are tight, and are of the order of 1%. As the coefficient of variance increases the gap in bounds increases, but the gap tends to decrease at high penalty cost for the similar reasons stated above.

	CV =0.1			CV=0.3		
Penalty Cost	C=1	C=4	C=12	C=1	C=4	C=12
p = 0.5	0.694	0.687	0.717	0.698	0.859	0.87
p=1	1.001	0.888	0.967	1.045	1.294	1.266
p=10	0.468	0.454	0.403	0.906	0.948	0.953

Table 4.2: Performance of Bounds when Demand is not Stationary

Table 4.3 exhibits the tightness of bounds when demand is stationary but retailers are non-identical. We allow for non-identical in retailers by varying coefficient of variation and the penalty costs across the retailers. In general, bounds are tight when retailers are non-identical in the penalty costs, but the gap widens if non-identicality is due to the difference in coefficient of variance.

CV/10	[1,1,1,1,1]	[1,1,1,3,3]	[1,1,1,1,1]	[1,1,1,3,3]	[1,1,1,3,3]
Penalty Cost	[1,1,1,5,5]	[1,1,1,5,5]	[1,1,1,10,10]	[1,1,1,1,1]	[1,1,1,10,10]
Spot Buy	81.1	53.7	83.18	45.11	63.33
Forward Buy	16.93	41.2	14.86	53.9	35.18
Order size	97.18	97.78	97.23	97.3	97.91
Warehouse Inventory	0.306	0.1722	0.2857	1.3	0.081
Total-Inventory	26.77	60.78	26.77	55.1	61
Upper Bound	95,828	96,232	95,908	95,378	96,473
Lower Bound	94,293	94,038	94,436	93,121	94,402
% Error	1.629	2.333	1.558	2.423	2.193

Table 4.3: Performance of Bounds when Retailers are Non Identical

4.6 Conclusions

This research contributes to the existing body of literature in the following ways 1) it models stochasticity of prices in a distributive supply chain 2) it elucidates that the market equilibrium information can be incorporated to obtain better procurement and distribution policies 3) it develops a method to bound a cost function for multi-location inventory model in presence of spot market. We obtain optimal policy for serial multi-echelon inventory system, and for multi-location distribution system the bounds developed are robust for non-stationary demand and non-identical retailers. This research addresses how market price information observed on the commodity market can be utilized to develop procurement and distribution policies to enhance the efficacy of the supply chain.

In particular, this research models the distribution of gasoline in a two echelon supply chain in a framework of one-warehouse multi-retailer which also has an additional procurement flexibility from the spot market. Researchers have struggled to find an optimal solution for the class of one-warehouse multi-retailer problems, therefore, they have resorted to develop bounds on the cost function. The proposed solutions in the literature are quite robust for identical retailers and stationary demand. Our paper extends the scope of the literature by developing a method that results in tighter bounds for the cases in which retailers are non-identical and demand is not stationary. Traditionally, in the problems of one-warehouse multi-retailer it is difficult to ascertain the cost of a unit commodity transferred from the warehouse to the downstream retailers due to the well known allocation constraint. This cost is primarily dependent on the characteristics of the retail network, and is difficult to compute, however, in our model cost of relaxing the allocation constraint can be determined through market equilibrium, and it is much simpler to compute. This cost is a combination of marginal convenience yield and transportation cost differential, hence, the information on marginal convenience yield facilitates the development of better procurement and distribution policies.

Our computational results derived from the model indicate that when demand variability is high it is optimal to procure more through forward contracts since central warehouse requires additional stocks to respond to the imbalances in inventory at downstream echelon caused due to high variability in demand. On the other hand, high uncertainty in price translates to higher futures prices which leads to the reduction in the cost of marginal convenience yield, which makes it less expensive to transact in the spot market leading to higher proportion of procurement from the spot market. Similarly, if the penalty cost incurred by downstream retailers is high then higher fraction of stock is procured from spot market as it is judicious to trade the cost of backlogging with the cost of transaction in spot market. The methodology proposed in this paper yields tighter upper bounds on cost functions when demand is non-stationary and retailers are non-identical. The main contribution of the paper is to highlight the importance of including market equilibrium information in deciding procurement and distribution policies.

There are several extensions possible of this work. In the literature of Operations Management considerable attention has been given to transshipment to optimize the inventory in one-warehouse multiple-retailer framework. It will be an interesting study to evaluate the conditions under which it makes strategic sense to procure from spot market to fulfill the immediate requirements for inventory versus having a transshipment from a different retailer. Also, the assumption of risk neutrality of a firm can be relaxed to incorporate risk-aversion of a firm, which may lead to explore the hedging strategies for optimal procurement.

Chapter 5

Effect of Term Structure of Futures Price on the Spot Procurement Policies

5.1 Introduction

Significant research has been done in the finance literature to develop pricing models that help predict the behavior of commodity prices. In general, these models are continuous time stochastic price models where prices are assumed to follow a Brownian Motion. These models involve the interaction among various factors that primarily influence the prices; these factors define the term structure of the futures price. It is common knowledge in the Finance literature that higher factor models are difficult to estimate using empirical data, but they provide a better fit to the futures price curve. However, it is not yet known that from the perspective of the operations manager how useful is the accuracy on future price estimates in determining its operating policies. In this research, we empirically test the significance of additional information on term structure of futures prices on the procurement and inventory related cost. In particular, we develop optimal procurement policy from the spot market, and compare this policy when futures price estimates are based on one factor model of Schwartz (1997), and two factor model of Schwartz and Smith (2000). We refer to these policies as non-adaptive policies because in these models the parameters of stochastic price process remains stationary. Furthermore, we allow for updating of stochastic prices process parameters for a two factor model in Schwartz and Smith (2000) and refer to these set of policies as adaptive policies and compare them against non-adaptive policies. In this research, we are using gasoline as a commodity of interest.

In past couple of years commodity prices have shown considerable fluctuations to significantly effect the procurement policies for commodity users. Price variation of commodity effects the contract price of a commodity traded between buyers and suppliers. There are contracts in practice that are subject to additional surcharge if the commodity price increases beyond a pre-specified limit. Fluctuations in commodity prices have generated a lot of interest in the finance literature which has led to the development of rich body of literature on commodity price models. In general, commodity prices exhibit cyclical behaviors which has motivated researchers to model commodity prices as mean reverting process. This implies that considerably high prices either leads to lower consumption of these commodities or increase in supply which drives prices towards its long term mean. Similarly, if prices decrease sufficiently then it prompts suppliers to withhold the supply of the commodity leading to an upward pressure on the prices. The fluctuation in prices reflect the dynamics of supply and demand of a commodity, and other factors related to political and economic conditions. Significantly, the objective of this research is to ascertain the impact of fluctuations in commodity prices on the procurement policies. In this research, we evaluate the relevance of analytically developed procurement policies by testing them through empirical data.

Historical data on commodity prices support the hypothesis of mean reversion in prices as empirically shown by Schwartz (1997) and Bessembinder et al. (1995). Bessembinder et al. (1995) finds strong mean reversion in agricultural commodities and crude oil, but the degree of mean reversion is less in metals. In contrast, Hjalmarsson (2003) shows that option prices for electricity based on Geometric Brownian Motion are more accurate in parameter estimation than the price process based on Ornstein-Uhlenbeck mean reverting process. This could be due to the absence of long-term traded contracts in electricity markets or due to the non-storable nature of electricity. Casassus and Collin-Dufresne (2005) document strong mean reversion in the price of oil and copper and mild mean reversion in the prices of Gold and Silver. In general, mean reversion in commodities is modeled as Ornstein-Uhlenbeck process such as described in Schwartz (1997) for single factor models. The two factor models include an additional model to explicitly model the short-term deviations from long-term equilibrium mean. Some examples of two factor models include Schwartz (1997), Schwartz and Smith (2000) and Gibson and Schwartz (1990). Two factor model provides a much better fit to the futures price curve than one factor model when there is more time variation in the marginal convenience yield of the commodity. Schwartz (1997), and Hilliard and Reis (1998) develop a three factor model to predict the commodity prices where the third factor captures the stochasticity in interest rates, and concludes that the the benefit of three factor model is marginal in comparison to that of two factor model. Thus, we limit our analysis to one and two factor models.

The challenge in implementing the stochastic price models is that the state variables of the term structure is not directly observable from the market data. Thus, it requires advanced statistical tools like Kalman filter to estimate the state variables and the parameters of the stochastic process. In addition, the parameters of stochastic process are not stationary and change with time, this confounds the problem of calibrating these stochastic price models. Stochasticity of volatility has been of concern in the Finance literature and has been addressed by Deng (2000), Pindyck (2004) and many other authors. Similarly, the non-stationarity in correlation coefficients has also been documented. One method of dealing with this issue is to model a parameter itself as a stochastic process, but such endeavor will lead to multiple factors in the model due to large number of parameters making impossible to develop a closed form solution for futures prices. Other solution to this problem is to calibrate the model more frequently as more information about futures price becomes available. We adopt this approach and update the parameters of the stochastic process on a monthly basis, and the optimal procurement policy developed is termed as adaptive policy. In this research, we compare the impact of adaptive policies with non-adaptive policies on the cost of procurement.

Economists have studied the impact of spread in spot and futures prices on the overall inventory levels of the industry. On the other hand, Operations Management has developed inventory policies for an individual firm based on constant or known prices. In this research, we combine the knowledge of these two areas to develop inventory policies for a firm under the paradigm of fluctuating prices, subsequently, these policies are empirically tested. In particular, we focus on procurement policies of a commodity from a spot market.

5.2 Term Structure Model

Commodity price models have been developed in Finance literature. In this paper, we compare the effect of one and two factor price process model on the procurement policies of a firm. We use one factor model as described in Schwartz (1997), and the two factor pricing model developed by Schwartz and Smith (2000), from now on referred to as S&S, for the evaluation of the commodity prices. These two models are described in detail below.

5.2.1 Schwartz (1997) One Factor Model

The one factor model follows a Ornstein Uhlenbeck process, which allows for mean reversion in prices as shown below

$$d\chi_t = \kappa(\alpha - \chi_t)dt + \sigma_\chi dZ \tag{5.1}$$

where, spot price of the commodity at time t is denoted by $ln(S_t) = \chi_t$, α represents the long term average of prices, κ is the mean reversion factor, σ_χ is the volatility associated with prices, and dZ is the increment of Brownian Motion associated with the stochastic process. Under a risk neutral measure, the futures price of a commodity given at time t with maturity of the futures contract at time T is given as

$$ln(f_{t,T}) = \chi_t(T-t) + \alpha^*(1 - e^{-\kappa(T-t)}) + \frac{\sigma_\chi^2}{4\kappa}(1 - e^{-2\kappa(T-t)})$$

where, α^* is given as $(\alpha - \lambda)$,where λ represents the risk premium of price of the commodity. For two factor model of Schwartz and Smith (2000) refer to Section 3.2.1.

5.3 Optimal Procurement Policy from a Commodity Market with Spot Transaction Costs.

In this section, we characterize optimal procurement policy for a firm that is either risk neutral to demand risk or is sufficiently capable to completely diversify the demand risk. This firm engages in spot procurement of the commodity, and transforms the basic commodity through its technological expertise to a value added product. At the start of each period, procurement manager evaluates the inventory position x_t and makes a spot procurement decision z_t in order to satisfy stochastic demand that follows a normal distribution $\phi_t(\xi)$. The unsatisfied demand is backlogged and the firm incurs a cost of p per unit per period for any unsatisfied demand. Similarly, excess inventory costs h per unit per period. These excess and shortage costs related to inventory can be summarized by Loss function given as

$$L_t(z_t) = \int_0^{z_t} h(z_t - \xi)\phi_t(\xi)\, d\xi + \int_{z_t}^{\infty} p(\xi - z_t)\phi_t(\xi)\, d\xi$$

The objective of the procurement manager is to minimize procurement and inventory costs in presence of stochastic prices and uncertain demand. The cost function is denoted as a dynamic program which has inventory level as the state variable. In addition, the dynamic program also depends on the variables associated with the price process; χ_t when using one factor model; χ_t and ω_t when using two factor model. However, for notational simplification we will drop the state variables of price from the dynamic program. The procurement model also allows for selling excessive stock of inventory in the spot market.

In practice, there is reluctance on the part of firms to frequently sell in commodity markets a short term excess inventory. This reluctance could be attributed to transaction costs, which consist of financial transaction cost as well as transportation and handling costs. Even though financial transaction costs can be considered negligible, handling and shipping goods from one location to another could be a costly affair; hence the importance of understanding the influence of these costs on the procurement process from spot markets. Thus, we include transaction costs in spot procurement. In order to understand better the dynamics induced by spot transaction costs we allow spot transaction costs incurred while selling and buying to be different. Let us denote the transaction cost per unit charged during buying and selling by λ_t^b and λ_t^s respectively, and their sum as $\lambda_t^{bs} = \lambda_t^b + \lambda_t^s$. At the end of the horizon, excess stock is sold in the spot market at the prevailing price in the market. Similarly, shortages are covered by buying in the spot market at current price. The costs associated with procurement is given by the following dynamic program

$$V_t^S(x) = \min_{z_t} \left\{ (S_t + \lambda_t^b)(z_t - x_t)^+ - (s_t - \lambda_t^s)(x_t - z_t)^+ + L_t(z_t) + H_t^S(z_t) \right\} \quad (5.2)$$

where,

$$H_t^S(z_t) = \beta \int_0^\infty \int_0^\infty \int_0^\infty V_{t+1}^S(z_t - \xi_t, \chi, \omega) \;\; \phi_t(\xi) \; \psi_t(\chi, \omega | \chi_t, \omega_t) \;\; d\xi \; d\chi \; d\omega \quad (5.3)$$

and, $\psi_t(\chi, \omega|\chi_t, \omega_t)$ is the probability density that the long term price factor and short term price factor in time $t+1$, assume the values (χ, ω) given that their values at time t are (χ_t, ω_t), and β is the discount rate corresponding to the risk free rate. The first term of the dynamic program given by (5.2) represent the cost of purchase from the spot market, the second term represents the revenues from selling excessive stock in the spot market, the third term gives the loss function that combines penalty and holding costs, and the fourth term refers to the cost to go function. Substituting $(z_t - x_t) = (z_t - x_t)^+ - (x_t - z_t)^+$ the above objective function can be modified as

$$V_t^S(x_t, \chi_t, \omega_t) \quad = \quad \min_{z_t} \left\{ (S_t + \lambda_t^b)(z_t - x_t) + \lambda_t^{bs}(x_t - z_t)^+ + L_t(z_t) + H_t^S(z_t) \right\};$$

where,

$$V_T^S(x_T, \chi_T, \omega_T) \quad = \quad (S_T + \lambda_T^b)(-x_T)^+ - (S_T - \lambda_T^s)(x_T)^+.$$

Define the function $J_t^S(x_t, z_t)$ as

$$J_t^S(x_t, z_t) = (S + \lambda_t^b)(z_t - x_t) + \lambda_t^{bs}(x_t - z_t)^+ + L_t(z_t) + H_t^S(z_t). \tag{5.4}$$

Lemma 5.1. Characterization of V_t^S and J_t^S.

The cost function $V_t^S(x_t, \chi_t, \omega_t)$ is convex in x_t, and $J_t^S(x_t, z_t)$ is convex in x_t and z_t.

Proof: Immediate by induction $\diamondsuit$.

Although J_t^S is convex, it can be observed from (5.4), that it is not differentiable at $z_t = x_t$, hence we need to rely on its sub-differential for its optimization. To this end denote as D^- and D^+ the left and right derivative respectively, we use $D^\pm$ as a shorthand to denote ‘both the left and right derivatives respectively, and denote the sub-differential of any convex function g at x with $\partial g(x)$ defined as the set valued function $\partial g \colon x \, [D_x^- g(x), D_x^+ g(x)]$ Rockafellar (1970). Then a sufficient condition for x^* to minimize $g(x)$ is that $0 \in \partial g(x^*)$. We will interpret scalar addition and positive scalar multiplication operations on sub-differentials and intervals as point-wise operations on each of its elements. Thus $a + b\partial J_t^S(z + c)$ will denote the interval $\left[a + bD_z^- J_t^S(z + c), a + bD_z^+ J_t^S(z + c)\right]$ for any scalars $a, b > 0$, and c. We also define the indicator function $1_{\{A\}}$ as assuming a value of 1 whenever the logical condition$\{A\}$ is true and zero otherwise. Theorem 5.1 below further refines the characterization of optimal procurement policies.

Theorem 5.1. Characterization of Optimal Procurement Policies.

The optimal procurement policy in spot market can be characterized by two points z_t^b *and* z_t^s, $z_t^b \leq z_t^s$ *as follows:*

(a) If $x_t < z_t^b$, $z_t^* = z_t^b$; *buy* $(z_t^b - x_t)$ *units to rise the commodity's inventory up to* z_t^b.

(b) If $x_t > z_t^s$, $z_t^* = z_t^s$; *sell* $(x_t - z_t^s)$ *units to lower the commodity's inventory down to* z_t^s.

(c) If $z_t^b \leq x_t \leq z_t^s$, $z_t^* = x_t$; *do not procure.*

Proof: we proceed by induction on t. It can be verified that the policy form is valid in period $T - 1$. For any other period t, we first assume the above policy form is valid for period $t + 1$, and consider the sufficient condition $0 \in \partial J_t^S(z_t)$ in the following cases: (1) $x_t < z_t^*$, (2) $x_t > z_t^*$, and (3) $x_t = z_t^*$. First we characterize the derivative of H_t^S. For any period t, we obtain from (5.3) and (5.4) $D_x^\pm V_{t+1}^S = D_x^\pm J_{t+1}^S + (D_{z^*}^\pm J_{t+1}^S)(D_x^\pm z^*)$ as

$$
\begin{aligned}
D_x^- V_{t+1}^S(x_{t+1}, \chi_{t+1}, \omega_{t+1}) &= -(S_{t+1} + \lambda_{t+1}^b) + (S_{t+1} + \lambda_{t+1}^b)\mathbf{1}_{\{z_{t+1}^b < x_{t+1} \le z_{t+1}^s\}} \\
&\quad + \lambda_{t+1}^{bs}\mathbf{1}_{\{x_{t+1} > z_{t+1}^s\}} \\
&\quad + \left[L'_{t+1}(x_{t+1}) + H_{t+1}^{S'}(x_{t+1})\right]\mathbf{1}_{\{z_{t+1}^b < x_{t+1} \le z_{t+1}^s\}} \\
D_x^+ V_{t+1}^S(x_{t+1}, \chi_{t+1}, \omega_{t+1}) &= -(S_{t+1} + \lambda_{t+1}^b) + (S_{t+1} + \lambda_{t+1}^b)\mathbf{1}_{\{z_{t+1}^b \le x_{t+1} < z_{t+1}^s\}} \\
&\quad + \lambda_{t+1}^{bs}\mathbf{1}_{\{x_{t+1} \ge z_{t+1}^s\}} \\
&\quad + \left[L'_{t+1}(x_{t+1}) + H'^S_{t+1}(x_{t+1})\right]\mathbf{1}_{\{z_{t+1}^b \le x_{t+1} < z_{t+1}^s\}}. \quad (5.5)
\end{aligned}
$$

It follows from (5.5) that $-(S_{t+1} + \lambda_{t+1}^b) \le D_x^{\pm} V_{t+1}^S \le -(S_{t+1} - \lambda_{t+1}^s)$. From the definition of H_t^S in (5.3) it follows that $D_{z_t}^{\pm} H_t^S(z_t) = \beta E_\xi E^{\mathbb{Q}}_{(\chi,\omega|\chi_t,\omega_t)} D_{z_t}^{\pm} V_{t+1}^S(z_t - \xi_t, \chi, \omega)$, and it follows from the risk neutral measure that $E^{\mathbb{Q}}_{(\chi,\omega|\chi_t,\omega_t)}(S_{t+1}) = f_t$; then, from (5.5) for any z_t we obtain $D_{z_t}^- H_t^S(z_t) = D_{z_t}^+ H_t^S(z_t) = H_t^{S'}(z_t)$ as

$$
H_t^{S'}(z_t) = -\beta(f_t + \lambda_{t+1}^b) + \beta\lambda_{t+1}^{bs}\,\mathrm{Pr}
$$

$$
\left[f_t + \lambda_{t+1}^b + E_\xi L'_{t+1}(z_t - \xi) + E_\xi H_{t+1}^{S'}(z_t - \xi)\right]\mathrm{Pr}\{z_{t+1}^b \le z_t - \xi_t < z_{t+1}^s\}
$$

and it follows from the convexity of V_{t+1}^S that $H_t^{S'}(z_t)$ is a nondecreasing function with $-\beta(f_t + \lambda_{t+1}^b) \le H_t^{S'}(z_t) \le -\beta(f_t - \lambda_{t+1}^s)$. The sub-differential $\partial J_t^S(z_t)$ is obtained from (5.4) and (5.6) as

$$
\begin{aligned}
\partial J_t^S(z_t) &= (S_t + \lambda_t^b) + \lambda_t^{bs}\left[-1_{\{z_t \le x_t\}}, -1_{\{z_t < x_t\}}\right] - p + (p+h)\Phi_t(z_t) \quad (5.7) \\
&\quad + H_t^{S'}(z_t).
\end{aligned}
$$

Case 1 ($x_t < z_t^*$): In this case, imposing the condition $0 \in \partial J_t^S(z_t)$ on (5.7) is equivalent to requiring

$$(S_t + \lambda_t^b) - p + (p+h)\Phi_t(z_t) + H_t^{S'}(z_t) = 0. \tag{5.8}$$

Observe that both $\Phi_t(z_t)$ and $H_t^{S'}(z_t)$ are nondecreasing functions, and that as $z_t {+}\infty$ the left hand side of the above condition becomes $S_t + h - f_t + (\lambda_t^b + \beta\lambda_{t+1}^s)$ which is positive as the marginal convenience yield cannot be negative. Define z_t^b as the value of z_t that solves the above condition or $z_t^b = 0$ if none does.

Case 2 ($x_t > z_t^*$): Imposing the condition $0 \in \partial J_t^S(z_t)$ on (5.7) results in

$$(S_t + \lambda_t^b) - p + (p+h)\Phi_t(z_t) + H_t^{S'}(z_t) = \lambda_t^{bs}. \tag{5.9}$$

An analysis similar to the one in *Case 1* leads to define z_t^s as the value of z_t that solves the above condition; we let $z_t^s = 0$ if the left hand side of the above condition is larger than $(\lambda_t^b + \lambda_t^s)$ for $z_t = 0$, or $z_t^s = +\infty$ if the limit of left hand side of the above condition is smaller than $(\lambda_t^b + \lambda_t^s)$ as $z_t + \infty$. The condition $z_t^b \leq z_t^s$ is also satisfied.

Case 3 ($x_t = z_t^*$): In this case, the first order condition emanating from (5.7) is

$$0 \leq (S_t + \lambda_t^b) - p + (p+h)\Phi_t(z_t) + H_t^{S'}(z_t) \leq \lambda_t^{bs}.$$

Comparing this condition with the ones for *Case 1* and *Case 2*, it follows that $z_t^* = x_t$ satisfies the above optimality condition for all $z_t^b \leq x_t \leq z_t^s \diamondsuit$.

The optimal policy for this model can be characterized as a *critical interval* policy in the sense that the optimal decision is to buy/sell enough stock so that the inventory position reaches the interval $[z_t^b, z_t^s]$, and do nothing if $x_t \in [z_t^b, z_t^s]$. Obtaining the values for z_t^b and z_t^s in the optimal policy is not practical as the evaluation of $\Pr\{z_t - \xi_t \geq z^s_{t+1}\}$ and $\Pr\{z^b_{t+1} \leq z_t - \xi_t < z^s_{t+1}\}$ in (5.6) is very complex. Notice that $\Pr\{z_t - \xi_t \geq z^s_{t+1}\}$ is the probability that the optimal policy in period $t+1$ requires to sell in the spot market while $\Pr\{z^b_{t+1} \leq z_t - \xi_t < z^s_{t+1}\}$ is the probability of not trading in $t+1$. Following we develop bounds on the optimal policy space

Theorem 5.2: Bounds on Optimal Procurement Policies

The optimal policy points z^b_t and z^s_t are bounded by $\underline{z}^b_t \leq z^b_t \leq \bar{z}^b_t$, and $\underline{z}^s_t \leq z^s_t \leq \bar{z}^s_t$, where the bounds can be computed myopically by solving:

$$\Phi_t(\underline{z}^b_t) = \frac{p + \beta f_t - \beta\lambda^s_{t+1} - S_t - \lambda^b_t}{p+h}, \qquad \Phi_t(\bar{z}^b_t) = \frac{p + \beta f_t + \beta\lambda^b_{t+1} - S_t - \lambda^b_t}{p+h},$$
$$\Phi_t(\underline{z}^s_t) = \frac{p + \beta f_t - \beta\lambda^s_{t+1} - S_t + \lambda^s_t}{p+h}, \qquad \Phi_t(\bar{z}^s_t) = \frac{p + \beta f_t + \beta\lambda^s_{t+1} - S_t + \lambda^s_t}{p+h}.$$

Proof: We develop upper bounds on z^b_t and z^s_t by solving (5.8) and (5.9) under the assumption that the optimal policy in $t+1$ requires us to procure with probability one, $\Pr\{z_t - \xi_t < z^b_{t+1}\} = 1$; similarly we obtain lower bounds on the optimal policy on period t by solving (5.8) and (5.9) under the assumption that in period $t+1$ the optimal policy will require us to sell with probability one, $\Pr\{z_t - \xi_t > z^s_{t+1}\} = 1\diamondsuit$.

The above bounds, or any point in between, can be used to approximate the optimal procurement policy; in particular, in Section 5.5, we use $\bar{z}^b_t \leq \bar{z}^s_t$ to define the policy $\tilde{z}_t$ as follows:

$$\tilde{z}_t = \begin{cases} \bar{z}^b_t, & \text{if } x_t < \bar{z}^b_t \\ x_t, & \text{if } \bar{z}^b_t \leq x_t \leq \bar{z}^s_t \\ \bar{z}^s_t, & \text{if } x_t > \bar{z}^b_t. \end{cases}$$

This approximation is equivalent to assuming that the probability of the optimal policy in $t+1$ requiring to buy stock in the spot market is one, $\Pr\{z_t - \xi_t < z^b_{t+1}\} = 1$.

We denote the cost of operating under this procurement policy as $\tilde{V}_t^S(x_t, \chi_t, \omega_t)$; this cost can be obtained from (5.2) inductively by replacing z_t with $\tilde{z}_t$ and eliminating the minimization operation. Clearly $V_t^S \leq \tilde{V}_t^S$. Like the model in Section 3.3.1, this approximate policy can be obtained myopically, and in order to test the tightness of such an approximation we develop a lower bound on the cost function. To this end define

$$\underline{V}_t^S(x_t, \chi_t, \omega_t) \;=\; \min_{z_t} \left\{ (S_t + \lambda_t^b)(z_t - x_t) + L_t(z_t) + \underline{H}_t^S(z_t) \right\}; \tag{5.10}$$

where

$$\underline{H}_t^S(z_t) \;=\; \beta \int_0^\infty \int_0^\infty \int_0^\infty \underline{V}_{t+1}^S(z_t - \xi_t, \chi, \omega) \;\; \phi_t(\xi) \; \psi_t(\chi, \omega | \chi_t, \omega_t) \;\; d\xi \; d\chi \; d\omega,$$

and

$$\underline{V}_T^S(x_T, \chi_T, \omega_T) \;=\; -(S_T + \lambda_T^b) x_T.$$

Thus, lower bound on the cost function $V_t^S(x_t, \chi_t, \omega_t)$ is obtained by ignoring the transaction cost of inventory sold in the spot market, $\lambda_t^{bs}(x_t - z_t)^+ \geq 0$.

Theorem 5.3: Lower bound on V_t^S.

(a) The function $\underline{V}_t^S$ defined by (5.10) is a lower bound on V_t^S.

(b) $\underline{V}_t^S$ is convex in x_t

Proof: To establish claim (a) We proceed by induction on t. It is immediate from their respective definitions that $\underline{V}_T^S \leq V_T^S$. Now assume the same relationship holds for $t + 1$; then $\underline{H}_t^S(z_t) \leq H_t^S(z_t)$, and since $(\lambda_t^b + \lambda_t^s)(x_t - z_t)^+ \geq 0$ we must have $\underline{V}_t^S \leq V_t^S$. The proof of claim (b) is immediate by induction on t.

We obtain from (5.10) that $D_{z_t}^+ \underline{H}_t^S(z_t) = -\beta f_t - \beta \lambda_{t+1}^b$. Then it follows from the convexity of $\underline{V}_t^S$ that the optimal policy for program (5.10), denoted as z_t^L, can be obtained myopically by solving

$$\Phi_t\left(z_t^L\right) = \frac{p + \beta f_t - S_t + \beta\lambda_{t+1}^b - \lambda_t^b}{p+h}.$$

It is clear from the analysis above that $\underline{V}_t^S \leq V_t^S \leq \tilde{V}_t^S$. Thus $\tilde{V}_t^S - \underline{V}_t^S$ is an upper bound on the approximation error incurred by using the approximate policy $\tilde{z}_t$ instead of the optimal. In Section 5.5 we calculate this approximation error.

5.4 Kalman Filter Procedure

Challenge in implementing the above mentioned pricing models is that the state variables and parameters of these models are not directly observable from the market data. Once the model is written in a state space form Kalman filtering process can be used to estimate the parameters of the process and the unobservable state variables based on the futures price data for the commodity. In this research, we are estimating the prices process parameters and state variables for gasoline. Kalman filtering is a statistical tool that recursively computes the optimal estimator of state variable based on the information available at time t. The estimates are continuously updated as new information about prices becomes available. If the initial state space variable distribution is normal then Kalman filter allows for the calculation of likelihood function to estimate the parameters of the prices process. Kalman filtering techniques are effective when either the initial state space distribution is well specified or the sample size is too large such that the initial state space specifications are unimportant. For detailed understanding on Kalman filtering we refer readers to Harvey (1991) and Schwartz and Smith (2000). Futures prices for different maturities are regressed with unobservable state variables through *measurement* equations.

State variables are generated through the *transition* equations, which discretized the stochastic process. These set of equations are described in detail for both pricing models in the following subsection.

5.4.1 One Factor Model

The measurement equation for one factor model can be written as

$$y_t = c_t + G_t X_t + \epsilon_t \qquad t = 1, 2, ...NT$$

where, $y_t = ln[f(T_i)]$ where i is the vector $N \times 1$ of observable prices. $c_t = [\alpha^*(1 - e^{-\kappa T_i}) + \frac{\sigma_\chi^2}{4\kappa}(1 - e^{-2\kappa T_i})]$; $T_1, T_2....T_n$ are the different times at which the different future contracts mature.

$G_t = [e^{-\kappa T_i}]$ where i is the vector $N \times 1$

ϵ_t is $N \times 1$ vector of serially uncorrelated disturbances, where $E(\epsilon_t) = 0$

From (5.1) the transition equation can be written as $X_t = d_t + Z_t X_{t-1} + \eta_t$

where $d_t = \kappa\alpha\Delta t$, $Z_t = e^{-\kappa\Delta t}$ and η_t is the vector of uncorrelated disturbances with $E(\eta_t) = 0$ and $Var(\eta_t) = \sigma_\chi^2 \Delta t$

5.4.2 Two Factor Model

The measurement equation for two factor model can be written as

$$y_t \;=\; c_t + G_t^{'} X_t + \epsilon_t$$

where, $y_t = ln[f(T_i)]$ where i is the vector $N \times 1$ of observable prices. $c_t = [A(T_i)]$ is a $N \times 1$ vector; and $G_t = [e^{-\kappa T_i} 1]$ is a matrix of $N \times 2$

ϵ_t is a vector $N \times 1$ of serially uncorrelated disturbances, where $E(\epsilon_t) = 0$;

From (3.1) and (3.2) the transition equation can be written as $X_t = d_t + Z_t X_{t-1} + \eta_t$ where, $d_t = \begin{bmatrix} 0 & \mu^* \Delta t \end{bmatrix}$ $Z_t = \begin{bmatrix} e^{-\kappa \Delta t} & 0 \\ 0 & 1 \end{bmatrix}$ is a 2×2 matrix, and η_t is the vector of uncorrelated disturbances with $E(\eta_t) = 0$ and
$Var(\eta_t) = \begin{bmatrix} (1 - e^{-2\kappa t})\frac{\sigma_\chi^2}{2\kappa} & (1 - e^{-\kappa t})\frac{\rho \sigma_\chi \sigma_\omega}{\kappa} \\ (1 - e^{-\kappa t})\frac{\rho \sigma_\chi \sigma_\omega}{\kappa} & \sigma_\omega^2 t \end{bmatrix}$

5.4.3 Parameter Estimation

We briefly discuss the parameter estimation process for a two factor model, but we refer readers to Schwartz and Smith (2000) for a detailed understanding of the parameter estimation process. With the set of transition and measurement equation, and observed futures prices for different maturities Kalman filter is run based on initial value of $[\chi_0, \omega_0]$. Initial mean vector m_0 and covariance matrix C_0 is assumed. In the following period, new observation on price in conjunction with the mean vector and covariance matrix of the previous period are used to calculate the posterior mean and covariance matrix. As stated by Schwartz and Smith (2000), this mean and covariance matrix is given as

$$E[\chi_t, \omega_t] = a_t + A_t(y_t - f_t)$$

$$Var[\chi_t, \omega_t] = C_t = R_t - A_t Q_t A_t^{'}$$

where,

$a_t = c_t + Z_t m_{t-1}$ mean of (χ_t, ω_t) based on what is known at time $t-1$

$R_t = Z_t C_{t-1} Z_t^{'} + W$; Covariance of (χ_t, ω_t) based on what is known at time $t-1$

$f_t = d_t + G_t^{'} a_t$ mean of futures price based on what is known at time $t-1$

$Q_t = G_t^{'} R_t G_t + V$ Covariance of period t futures price based on what is known at time $t-1$

$A_t = R_t G_t Q_t^{-1}$; Correction to predicted state variables a_t based on the difference in actual and predicted value of futures price

As stated in Harvey (1991) Kalman filtering allows the calculation of a likelihood function for a given set of parameters. In the case of two-factor model of SS there are seven parameters to be estimated $(\kappa, \sigma_\chi, \sigma_\omega, \rho, \mu, \mu^*, \lambda_\chi)$, along with terms in the covariance matrix for the measurement errors (V). As mentioned in Schwartz (1997), Schwartz and Smith (2000) and Hahn (2005) it is assumed for simplification that the errors in the covariance matrix are uncorrelated. The general form for the log-likelihood function for a joint normal distribution is given as

$$ln(L) \quad = \quad -\frac{1}{2}\sum_{t=1}^{T} ln|F_t| - \frac{1}{2}\sum_{t=1}^{T}(y_t - \hat{y}_{t|t-1})^{'} F_t^{-1}(y_t - \hat{y}_{t|t-1}) + const$$

where,

$F_t = E[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})^{'}]$, Schwartz and Smith (2000) use maxlik routine in Gauss to numerically determine the estimates of the above parameters. In order to estimate the parameters a set of input parameters are provided then Kalman filter is run that outputs the optimal set of parameters, which might not be global. Global optimal parameters are the once that give maximum likelihood value. Thus, different set of starting parameters are used to search for the set of global optimal parameters.

5.5 Empirical Results

We use data on futures contract for unleaded Gasoline HU traded on NYMEX. The prices are listed in US cents for one gallon of gasoline. Our data set is from 10/1/99 to 3/8/2005 for contracts maturing from 5 months to up to 11 months. Gasoline price are known to exhibit seasonality, however, due to non-availability of long term maturing futures contracts it is difficult to accurately estimate the seasonality component in the prices. Therefore, we refrain from estimating seasonality in prices in this research. Next, we show the results of parameter estimation for one and two factor models.

5.5.1 State Variable and Parameter Estimation

Table 5.1 shows the parameter estimation for the one factor model. We observe that the mean reversion in prices is slow, and volatility in prices is lower than in comparison to that estimated from two factor model. Parameter estimates from the two factor model is listed in Table 5.2. These estimates are based on monthly basis from March 2004 to February 2005, where parameters are estimated till March 2004 using the data set from 10/1/99 to 3/31/2004 and thereafter, these parameters are updated on the monthly basis. In addition, the standard error associated with these estimates are also listed in Table 5.2. The short-term volatility in prices is much higher than in comparison to the long-term equilibrium price volatility. Also, we observe that the volatilities for short term and long term factor remains almost constant over a year. The mean reversion factor fluctuates somewhat between 1.63 and 1.89. Typically, mean reversion factor is difficult to estimate as there are only limited number of cycles of price over a given data set of prices. Interestingly, we find negative correlation between short-term and long-term prices. The correlation factor is small in magnitude and the standard error associated with its estimation is large so it is not clear if the correlation factor is significant. However, this observation is puzzling and we do not have a justifiable explanation for this phenomenon at this time. Furthermore, on examining the standard errors for the parameters we observe that the estimates for μ and λ_χ are not very accurate due to larger values of standard error. We do get a good estimate of $\mu^* = \mu - \lambda_\omega$, which represents the risk-adjusted long-term growth/decline in the futures prices. The explanation for inaccurate estimation of μ and λ_χ can be attributed to Schwartz and Smith (2000) who conclude that λ_χ and λ_ω determines the difference in expected spot and futures prices, and since expected spot prices are never observable it is difficult to estimate the parameters related to risk premium. On the other hand, the state variables are easy to estimate with almost negligible error. We estimate initial state parameters as $\chi_0 = 4.34$ and $\omega_0 = -0.045$.

Parameter	Estimation	Standard Error
α	5.916	0.0096
κ	0.086	0.0025
σ	0.23	0.0138
σ^*	2.59	0.0083

Table 5.1: Estimation of parameters from One Factor Model

Parameters	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb
Estimated												
μ	0.070	0.080	0.101	0.098	0.098	0.125	0.130	0.155	0.150	0.143	0.153	0.158
κ	1.636	1.659	1.791	1.780	1.780	1.752	1.683	1.713	1.748	1.849	1.875	1.893
λ_X	0.351	0.366	0.399	0.428	0.428	0.453	0.461	0.458	0.454	0.441	0.450	0.448
σ_X	0.391	0.388	0.379	0.390	0.390	0.390	0.397	0.389	0.385	0.383	0.381	0.379
σ_ω	0.163	0.162	0.161	0.160	0.160	0.162	0.167	0.169	0.168	0.168	0.167	0.167
ρ	-0.149	-0.128	-0.035	-0.070	-0.070	-0.086	-0.141	-0.115	-0.090	-0.016	0.002	0.012
μ^*	-0.101	-0.099	-0.095	-0.080	-0.080	-0.071	-0.063	-0.071	-0.075	-0.084	-0.087	-0.089
Standard Error												
μ	0.077	0.075	0.075	0.016	0.016	0.074	0.077	0.078	0.076	0.071	0.073	0.075
κ	0.162	0.163	0.158	0.155	0.155	0.152	0.157	0.142	0.153	0.149	0.155	0.143
λ_X	0.085	0.089	0.091	0.090	0.090	0.085	0.090	0.091	0.090	0.097	0.092	0.088
σ_X	0.031	0.028	0.028	0.030	0.030	0.026	0.030	0.029	0.027	0.026	0.026	0.026
σ_ω	0.015	0.013	0.013	0.013	0.013	0.012	0.014	0.013	0.012	0.011	0.012	0.012
ρ	0.140	0.146	0.132	0.131	0.131	0.137	0.135	0.122	0.125	0.110	0.122	0.114
μ^*	0.016	0.015	0.015	0.015	0.015	0.015	0.017	0.016	0.015	0.013	0.013	0.013

Table 5.2: Monthly Parameter Estimation for Two Factor Model

5.5.2 Model Comparison

In this section, we compare three policies based on optimal policy for the lower bound developed in Theorem 5.3 in section 5.4. First policy is termed as One Factor model (OF) in which future price estimates are based on the one-factor model and the spot price observed in this period as discussed in Section 5.2.1. Similarly, second policy is termed as Non -Adaptive Two-Factor model (NATF) where future price estimates are based on a two factor model, as discussed in section 3.2.3, where parameters of the model are not update and remain stationary for the rest of the horizon. Third policy is termed as Adaptive Two Factor model (ATF) where parameters of the stochastic process are updated on the monthly basis upon the availability of new information on futures prices. In this analysis, the stochastic process follows that adaptive process acting as a benchmark to judge the performance of the OF and NATF models. In order to have a detailed analysis, we break procurement cost into two parts, controllable and uncontrollable cost. Uncontrollable cost are the cost that are unaffected by the procurement policy. Controllable costs are further broken down into penalty costs, holding costs, trading costs and Appreciation/Depreciation of inventory. The cost function for a procurement model can be written as

$$\begin{aligned} C(x_t) &= \sum_{t=0}^{N} \Big[(S_t + \lambda_t^b)(z_t - z_{t-1} + \xi_{t-1})^+ - (S_t - \lambda_t^s)(z_{t-1} - z_t - \xi_{t-1})^+ \\ & \quad + h(z_t - \xi_t)^+ + p(\xi_t - z_t)^+, \end{aligned}$$

where $x_t = z_{t-1} - \xi_{t-1}$; through algebraic manipulation this cost function can further be re-written as

$$\begin{aligned} C(x_t) &= \sum_{t=0}^{N} \Big[(S_t + \lambda_t^b - S_{t+1} - \lambda_{t+1}^b)(z_t - \xi_t) + h(z_t - \xi_t)^+ \\ & \quad + p(\xi_t - z_t)^+ + (\lambda_t^b + \lambda_t^s)(z_{t-1} - z_t - \xi_{t-1})^+ \Big] + \sum_{t=0}^{N} (S_t + \lambda_t^b)\xi_t \end{aligned} \tag{5.11}$$

where, $\xi_{t-1} = 0$. First term in (5.11) represents Appreciation/ Depreciation cost of inventory, second term is the holding cost, third term is the penalty cost, fourth term represents the trading costs. These first four terms comprise controllable cost. The fifth term is the uncontrollable cost. Notice, that Appreciation/Depreciation cost of inventory captures the change in valuation of leftover inventory from one period to another. All other costs are self-explanatory. Comparative analysis between various policies is done based on the controllable costs. Δ_{A-N} denotes the benefit of Adaptive policy (ATF) over Non-Adaptive (NATF) policy, Δ_{A-O} represents the benefit of Adaptive policy (ATF) over One Factor (OF) model, and Δ_{N-O} represents the benefits of Non-Adaptive (NATF) policy over One Factor (OF) model. Table 5.3 illustrates the comparative analysis of the policies, and the overall conclusion is that policies based on two factor model outweighs the policy based on one factor model. Observe that the stocking policy based on One Factor (OF) model yields higher stocking levels than in comparison to NATF based policy. This allows OF based policy to have a lower penalty costs in compare to NATF policy, but the real cost advantage of NATF policy is derived from Appreciation/Depreciation costs of inventory. Better ability to exploit the time varying marginal convenience yield allows NATF based policy to achieve lower costs in-spite of carrying lower inventory. This can have a profound impact in supply chains and to the financial performance of the firms as lower inventory levels can free extra cash to be used for more innovative purposes. When transaction cost is low the benefits of two factor model over single factor model are around 27%, but with increase in transaction costs these benefits decline to around 13%. Two factor model captures the time variation in convenience yield much better than the one factor model which leads to more efficient cost management under two factor model paradigm. Surprisingly, there is no benefit of ATF based policy over NATF policy. ATF based policy have even lower stocking levels than NATF based policy, thus, higher penalty costs, but these extra costs are equally balanced out by saving from the Appreciation/Depreciation costs. Interestingly, the error in bounds is negligible when coefficient of variation is small and transaction costs are small. However, the gap in the bound increases sharply with higher transaction costs and larger coefficient of variation. Thus, the policy developed in the lower bound is not robust for the high transaction cost and high demand variance environment.

	CV=0.2				CV=0.4			
	1.000	4.000	10.000	15.000	1.000	4.000	10.000	15.000
One Factor								
Trading Cost	0.000	0.000	0.000	0.000	3.984	15.936	39.841	59.762
Penalty Cost	66.630	67.399	68.937	70.231	133.273	134.798	137.875	140.462
Holding Cost	82.580	81.888	80.504	79.365	165.177	163.776	161.007	158.729
ADP Cost	25.280	24.792	23.961	23.216	50.365	49.584	47.921	46.432
Lower Bound	174.490	174.079	173.402	172.812	348.815	348.158	346.804	345.623
Upper Bound	174.490	174.079	173.402	172.812	352.799	364.095	386.644	405.385
% Error	0.000	0.000	0.000	0.000	1.142	4.577	11.488	17.291
Two Factor								
Trading Cost	0.230	0.948	2.442	3.737	4.821	19.245	48.829	72.796
Penalty Cost	171.700	173.563	177.538	180.888	316.985	320.018	326.239	331.370
Holding Cost	46.410	45.937	44.982	44.197	92.854	91.890	89.980	88.412
ADP Cost	-92.720	-94.510	-98.844	-102.358	-152.833	-155.956	-161.874	-167.025
Lower Bound	125.390	124.990	123.676	122.727	257.006	255.951	254.345	252.758
Upper Bound	125.620	125.938	126.117	126.464	261.828	275.197	303.174	325.553
% Error	0.183	0.759	1.974	3.045	1.876	7.519	19.198	28.801
Adaptive								
Trading Cost	0.486	1.948	4.980	7.635	5.590	22.528	56.642	85.155
Penalty Cost	185.970	188.058	192.439	196.233	334.592	337.790	344.240	349.666
Holding Cost	45.190	44.724	43.789	43.022	90.415	89.472	87.603	86.069
ADP Cost	-105.710	-107.885	-112.700	-116.723	-167.906	-171.007	-177.314	-182.484
Lower Bound	125.450	124.897	123.527	122.531	257.101	256.255	254.528	253.251
Upper Bound	125.936	126.845	128.508	130.166	262.691	278.783	311.171	338.406
% Error	0.387	1.560	4.032	6.231	2.174	8.791	22.254	33.625
Comparison								
% ATF Over NATF	-0.001	-0.006	-0.012	-0.016	-0.003	-0.011	-0.018	-0.024
% ATF Over OF	0.264	0.226	0.173	0.142	0.246	0.204	0.145	0.111
% NATF Over OF	0.266	0.230	0.182	0.154	0.248	0.212	0.161	0.133

Table 5.3: Comparative Analysis of One Factor, Two Factor and Adaptive-Two Factor Based Policies

5.5.3 Cyclical Demand

Policy obtained through lower bound on cost function yields sufficiently tight bounds when transaction costs are lower, but we seek to explore how it performs when demand is non-stationary, We consider demand as non-stationary and cyclical that follows a sine curve given by $\xi_t = \mu_t + A * sin\{c * \pi * (t-1)/24.5\}$, where A is the amplitude and C is the frequency of the sine curve. Table 5.4 illustrates that bound developed on cost function is not tight when demand is highly cyclical and follows almost a monthly cycle. The quality of bound further deteriorates when coefficient variation is high and the Amplitude of the demand curve is high.

	CV=0.2			CV=0.4		
Amplitude	C=1	C=4	C=12	C=1	C=4	C=12
5	0.99	0.96	0.967	9.873	9.814	11.07
10	1.036	0.976	1.334	10.33	10.55	16.24
20	1.122	1	3.307	11.38	13.74	41.268

Table 5.4: % Error on Bounds for the Cyclical Demand Based on Lower Bound Policy

The % error in the bound is 41% when coefficient of variation is 0.4 and demand curve follows a monthly cycle with amplitude equal to 40% of the mean demand. Therefore, we propose to use a $[\bar{z}_t^a, \bar{z}_t^b]$ policy described in Section 5.3 . This policy is a *critical interval* policy which restricts the selling in the spot market and leads to much tighter bounds. Table 5.5 shows that the gap in bound decreases from 41% to 1.13 % on implementation of the two point policy, and the quality of bound is good for high variance in cyclical demand with high amplitude in demand curve. In addition, Table 5 .6 illustrates that bounds are also very tight when transaction costs are high for highly variable demand.

	CV=0.2			CV=0.4		
Amplitude	C=1	C=4	C=12	C=1	C=4	C=12
5	0.0118	0.046	0.062	0.069	0.35	0.44
10	0.013	0.049	0.06	0.0742	0.36	0.32
20	0.0152	0.056	0.0563	0.0851	0.38	0.283

Table 5.5: % Error on bounds for the cyclical demand based on Two Point policy

Transaction Cost	CV=0.2			CV=0.4		
	C=1	C=4	C=12	C=1	C=4	C=12
1	0.003	0.0112	0.015	0.0146	0.08	0.103
4	0.0095	0.0382	0.053	0.0575	0.295	0.364
8	0.0181	0.066	0.084	0.102	0.513	0.62
15	0.03	0.0934	0.118	0.166	0.74	0.83

Table 5.6: % Error on bounds for the cyclical demand based on Two Point policy

5.6 Conclusions

We explore the procurement policy of a commodity from the spot market under stochastic demand and random prices for a multi-period planning horizon. We characterize the optimal procurement policy and bound the optimal policy space. This research addresses the impact of information on term structure of futures prices on the procurement policy of an operations manager. There is volumes of literature in finance documenting one factor and two factor based commodity pricing models. Yet, there has not been a study that delves into exploring the impact of these models on the operational policy of a firm. We conclude that two factors based futures price forecast yields substantial improvements in costs over the policy that has futures price forecast based on a single factor. In addition, for gasoline we do not observe any advantage of calibrating the two factor model more frequently as the parameters of stochastic price process remain almost stationary for the one year planning horizon. A better futures price forecast allows a firm to stock less and yet reduce the cost of inventory by capitalizing on the time varying marginal convenience yield. Inventory reduction is a prime focus of any firm in order to enhance flexibility and agility to respond better to the changing dynamics of the marketplace. The policy developed in this research can lead to better operational efficiency of a firm and enhance its financial performance.

Chapter 6

Concluding Remarks and Future Research

Commodity prices have shown considerable fluctuation in past couple of years. Operations Managers confront the problem of fluctuating prices when designing the optimal procurement policies. This research provides a framework to incorporate information of prices on spot and future markets in obtaining optimal procurement and distribution policies. Specifically, this research achieves four main objectives 1) models stochasticity in prices through a continuous stochastic price process 2) incorporates information on time varying marginal convenience yield 3) characterizes optimal procurement and distribution policies for various supply chain structures, and 4) quantifies the benefits of using a two factor model based futures price forecast in cost management.

The main managerial insight from this research postulates that commodity market exogenously determines the cost of holding a commodity through spread in spot and futures prices also referred to as marginal convenience yield, on the other hand, a firm has a decreasing marginal benefit of each additional unit of commodity stored. The balancing of these costs and benefits yields optimal stocking levels. In addition, these trade-off change every period which requires to adjust stocking levels accordingly. This entails to incorporating the time varying marginal convenience yield in the analysis. The central theme in the dissertation is to ascertain how firms should adapt their operating policies in wake of changing dynamics of supply and demand that gets reflected in the prices. Our results illustrate that on incorporating time varying convenience yield in the structure of the optimal policy can reduce inventory related costs substantially. Inventory reduction is a pursuit of every firm, achieving this objective without sacrificing the flexibility to satisfy random demand improves the financial performance of the firm and makes supply chain more efficient. This research develop tools that specifically will allow operation managers to achieve these objectives.

Furthermore, in the third chapter of dissertation, we develop buying and selling policies from spot and forward markets in presence of transaction costs. We are able to characterize the policy structure but it is not easy to compute it, thus, we develop an approximation of the policy and show that bounds on the cost function are tight. Forward procurement policies are characterized by a critical interval, where if the inventory is below the lower end of the critical interval than forward purchase is made upto the lower end of the critical interval, if the inventory is higher than the upper end of the critical inventory than inventory is brought down to the upper end of the critical interval by selling excesses in the forward market. Spot market inventory policy has a similar yet more complicated structure to it. Moreover, results indicate that manufacturers are willing to trade inferior demand information with lower transaction cost to procure higher fraction from the forward market and procure from spot market to fine tune the stocking levels.

In addition, we elaborate in the fourth chapter of dissertation how marginal convenience yield can be used to design procurement and distribution policies in a multi-echelon distributive system. We develop bounding techniques for the Lagrange multiplier of an allocation constraint to obtain efficient procurement and distribution policies without making a balancing assumption, a common assumption in the one-warehouse multiple retailers related problem. The dualization technique developed in this chapter allows us to develop tight bounds even when demand is non-stationary and retailers are non-identical.

Once we develop operating policies based on the forecast of futures prices, we quantify the benefits of using two-factor model over one-factor model. One-factor and two-factor models are well researched in the Finance literature and it has been well established that two-factor models provide a better fit to the forecast of futures prices. However, it is not known how much value a two-factor model creates for an operations manager. Our empirical analysis on gasoline shows that there are substantial cost savings on using a two-factor model to forecast the futures price. In addition, the parameters of stochastic price process do not remain stationary and there arises a need to frequently calibrate the models. Our analysis illustrates that for gasoline there is negligible benefit for calibrating the stochastic price process frequently.

APPENDIX A

Proof Theorem 3.2: We proceed by induction on t. It can be verified that the policy form is valid in period $T-1$. For any other period t, we first assume the above policy form is valid for period $t+1$, and verify that the sufficient conditions $0 \in \partial_z J_t^S(z_t^*, y_t^*)$ and $0 \in \partial_y J_t^S(z_t^*, y_t^*)$ are satisfied in all cases (a) -(h). The proof proceeds by verifying that $0 \in \partial_z J_t^{SF}(z_t^*, y_t^*)$ for z_t^* as specified in each of the five parts (d) through (h), and in each of the above cases we show that $0 \in \partial_y J_t^S(z_t^*, y_t^*)$ for the values of y_t^* prescribed by parts (a)-(c) of the theorem.

Part (a): If $z_t \leq y_t^b$ then $y_t^* = y_t^b$, thus we need to show that $0 \in \partial_y J_t^{SF}(z_t, y_t^b)$. It follows from (3.19) that $\partial_y J_t^{SF}(z_t, y_t^b) = [-\beta\alpha_t^{bs}, 0]$, hence $0 \in \partial_y J_t^{SF}(z_t, y_t^b)$

Part (b): If $y_t^b \leq z_t \leq y_t^s$, then $y_t^* = z_t$, thus we need to show that $0 \in \partial_y J_t^{SF}(z_t, z_t)$. $\partial_y J_t^{SF}(z_t, y_t^b) = \beta[f_t + \alpha_t^b + H_t^{SF'}(z_t) - \beta\alpha_t^{bs}, f_t + \alpha_t^b + H_t^{SF'}(z_t)]$. Sub-differential $\partial_y J_t^{SF}(z_t, z_t)$ goes from $[-\beta\alpha_t^{bs}, 0]$ for $y_t^* = y_t^b$ to $[0, \beta\alpha_t^{bs}]$ for $y_t^* = y_t^s$. Hence, $0 \in \partial_y J_t^{SF}(z_t, z_t)$.

Part (c): If $y_t^s \leq z_t$ then $y_t^* = y_t^s$ thus we need to show that $0 \in \partial_y J_t^{SF}(z_t, y_t^s)$. It follows from (3.20) that $\partial_y J_t^{SF}(z_t, y_t^s) = [0, \beta\alpha_t^{bs}]$, hence $0 \in \partial_y J_t^{SF}(z_t, y_t^s)$

Part (d): We need to consider the following four cases depending of the values of y_t^b and y_t^s.

Case (d-*i*) $y_t^b \leq y_t^s < z_t^b$: In this case $z_t^* = z_t^b$, and $y_t^* = y_t^s < z_t^b$, thus we need to show that $0 \in \partial_z J_t^{SF}(z_t^b, y_t^s)$, and $0 \in \partial_y J_t^{SF}(z_t^b, y_t^s)$. It follows from (3.14) that the first order condition $0 \in \partial_z J_t^{SF}(z_t^b, y_t^s)$ becomes $K_t(z_t^b) = -\beta\alpha_t^{bs}$ as claimed in (3.15). If $K_t(0) \leq -\beta\alpha_t^{bs}$, since K_t is nondecreasing, there is a z_t satisfying the above condition; if $K_t(0) > -\beta\alpha_t^{bs}$, we define $z_t^b = 0$. Moreover, from (3.13) it follows that the condition $0 \in \partial_y J_t^{SF}(z_t^b, y_t^s)$ is equivalent to $H_t^{SF'}(y_t^s) = \alpha_t^s - f_t$ as in (3.20).

Case (d-*ii*) $y_t^b < z_t^b \leq y_t^s$: Now $z_t^* = z_t^b = y_t^*$, and we need to show that

$0 \in \partial_z J_t^{SF}(z_t^b, z_t^b)$, and $0 \in \partial_y J_t^{SF}(z_t^b, z_t^b)$. It follows directly from (3.14) and (3.15) that $\partial_z J_t^{SF}(z_t^b, z_t^b) = [-\beta\alpha^{bs}, 0]$, and from (3.13) we obtain $\partial_y J_t^{SF}(z_t^b, z_t^b) = \beta[f_t - \alpha_t^s + H_t^{SF'}(z_t^b), f_t + \alpha_t^b + H_t^{SF'}(z_t^b)]$. It follows from (3.19) and (3.20) that $-f_t - \alpha_t^b \leq H_t^{SF'}(z_t^b) \leq -f_t + \alpha_t^s$ for $y_t^b \leq z_t^b \leq y_t^s$, hence $\partial_y J_t^{SF}(z_t^b, z_t^b)$ ranges from $\beta[-\alpha_t^{bs}, 0]$ to $\beta[0, \alpha_t^{bs}]$ as z_t^b goes from y_s^b to y_t^s, hence $0 \in \partial_y J_t^{SF}(z_t^b, z_t^b)$.

Case (d-*iii*) $z_t^{cb} < y_t^b$: In this case, $z_t^* = z_t^{cb}$ and $y_t^* = y_t^b$. The first order conditions become $\partial_z J_t^{SF}(z_t^{cb}, y_t^b) = K_t(z_t^{cb}) = 0$ as claimed in (3.16), and $\partial_y J_t^{SF}(z_t^{cb}, y_t^b) = \beta\left(f_t + \alpha_t^b + H_t^{SF'}(y_t^b)\right) = 0$ is equivalent to $H_t^{SF'}(y_t^b) = -f_t - \alpha_t^b$ establishing (3.19). Hence $z_t^* = z_t^{cb}$ and $y_t^* = y_t^b$ minimize J_t^{SF}. Moreover, since both $K_t(z_t)$ and $H_t^{SF'}(y_t)$ are nondecreasing, it follows that $z_t^b \leq z_t^{cb}$ and $y_t^b \leq y_t^s$.

Case (d-*iv*) $z_t^b \leq y_t^b \leq z_t^{cb}$: Here (a) and (d) prescribe $z_t^* = y_t^* = y_t^b$ and the optimality conditions become $0 \in \partial_z J_t^{SF}(y_t^b, y_t^b) = \left[K_t(y_t^b), K_t(y_t^b) + \beta\alpha_t^{bs}\right]$ and the condition $0 \in \partial_y J_t^{SF}(y_t^b, y_t^b)$ becomes $0 \in \left[f_t - \alpha_t^s + H_t^{SF'}(y_t^b), f_t + \alpha_t^b + H_t^{SF'}(y_t^b)\right]$. It follows from the analysis of cases (d-i) and (d-ii) that the sub-differential $\partial_z J_t^{SF}(y_t^b, y_t^b)$ goes from $[-\beta\alpha_t^{bs}, 0]$ for $y_t^b = z_t^b$ to $[0, \beta\alpha_t^{bs}]$ for the case $y_t^b = z_t^{cb}$, hence $0 \in \partial_z J_t^{SF}(y_t^b, y_t^b)$. It follows from (3.19) that in this case $0 \in \partial_y J_t^{SF}(y_t^b, y_t^b) = \beta[-\alpha_t^{bs}, \alpha_t^{bs}]$.

Part (e): Again consider the following four cases.

Case (e-*i*) $y_t^s < x_t$: Now the policy prescribes $z_t^* = x_t$ and $y_t^* = y_t^s$, and the sub-differential $\partial_z J_t^{SF}(x_t, y_t^s) = \left[K_t(x_t) + \beta\alpha_t^{bs} - \lambda_t^{bs}, K_t(x_t) + \beta\alpha_t^{bs}\right]$. It follows from (3.15) and (3.16) that this sub-differential ranges from $[-\lambda_t^{bs}, 0]$ for $x_t = z_t^b$ to $[\beta\alpha_t^{bs} - \lambda_t^{bs}, \beta\alpha_t^{bs}]$ for $x_t = z_t^{cb}$; hence $0 \in \partial_z J_t^{SF}(x_t, y_t^s)$ for all $z_t^b \leq x_t \leq z_t^{cb}$. Similarly, $\partial_y J_t^{SF}(x_t, y_t^s) = \beta\left(f_t - \alpha_t^s + H_t^{SF'}(y_t^s)\right)$; it follows from (3.20) that this sub-differential is zero.

Case (e-*ii*) $y_t^b \leq x_t \leq y_t^s$: In this case $z_t^* = y_t^* = x_t$ and the sub-differential $\partial_z J_t^{SF}(x_t, x_t) = \left[K_t(x_t) - \lambda_t^{bs}, K_t(x_t) + \beta\alpha_t^{bs}\right]$. It follows from cases (3.15) and

(3.16) that this sub-differential ranges from $[-\beta\alpha_t^{bs} - \lambda_t^{bs}, 0]$ for $z_t^* = x_t = z_t^b$ to $[-\lambda_t^{bs}, \beta\alpha_t^{bs}]$ for $x_t = z_t^{cb}$; hence $0 \in \partial_z J_t^{SF}(x_t, x_t)$ for all $z_t^b \leq x_t \leq z_t^{cb}$. Similarly, $\partial_y J_t^{SF}(x_t, x_t) = [f_t - \alpha_t^s + H_t^{SF'}(x_t), f_t + \alpha_t^b + H_t^{SF'}(x_t)]$, and from (3.19) and (3.20) it follows that $-f_t - \alpha_t^b \leq H_t^{SF'}(x_t) \leq -f_t + \alpha_t^s$ for $y_t^b \leq x_t \leq y_t^s$, hence $\partial_y J_t^{SF}(x_t, x_t)$ ranges from $[-\alpha_t^{bs}, 0]$ to $[0, \alpha_t^{bs}]$ as x_t goes from y_s^b to y_t^s, hence $0 \in \partial_y J_t^{SF}(x_t, x_t)$.

Case (e-*iii*) $x_t < y_t^b \leq z_t^{cb}$**:** Now $z_t^* = y_t^* = y_t^b > x_t$, and $\partial_z J_t^{SF}(y_t^b, y_t^b) = \left[K_t(y_t^b), K_t(y_t^b) + \beta\alpha_t^{bs}\right]$. This sub-differential ranges from $[-\beta\alpha_t^{bs}, 0]$ for $x_t = z_t^b$ to $[0, \beta\alpha_t^{bs}]$ for $x_t = z_t^{cb}$, thus guaranteeing that $0 \in \partial_z J_t^{SF}(y_t^b, y_t^b)$. Similarly $\partial_y J_t^{SF}(y_t^b, y_t^b) = \beta[f_t - \alpha_t^s + H_t^{SF'}(y_t^b), f_t + \alpha_t^b + H_t^{SF'}(y_t^b)] = \beta[-\alpha_t^{bs}, 0]$, hence $0 \in \partial_z J_t^{SF}(y_t^b, y_t^b)$.

Case (e-*iv*) $x_t < z_t^{cb} < y_t^b$**:** Now $z_t^* = z_t^{cb}$ and $y_t^* = y_t^b > z_t^*$. It follows from (3.16) that $\partial_z J_t^{SF}(z_t^{cb}, y_t^b) = K_t(z_t^{cb}) = 0$, and from (3.19) it follows that $\partial_y J_t^{SF}(z_t^{cb}, y_t^b) = \beta\left(f_t + \alpha_t^b + H_t^{SF'}(y_t^b)\right) = 0$.

Part (h): Consider the following four cases.

Case (h-*i*) $z_t^s < y_t^b$**:** In this case $z_t^* = z_t^s < y_t^* = y_t^b$, then from (3.19) we obtain $\partial_y J_t^{SF}(z_t^s, y_t^b) = \beta\left(f_t + \alpha_t^b + H_t^{SF'}(y_t^b)\right) = 0$, and $\partial_z J_t^{SF}(z_t^s, y_t^b) = K_t(z_t^s) - \lambda_t^{bs}$. Hence z_t^s is a solution to $K_t(z_t^s) = \lambda_t^{bs}$ as claimed in (3.18)

Case (h-*ii*) $z_t^s < y_t^s$**:** Now the theorem indicates $z_t^* = y_t^* = z_t^s$, and it follows from (3.18) that $\partial_z J^{SF}(z_t^s, z_t^s) = [K_t(z_t^s) - \lambda_t^{bs}, K_t(z_t^s) + \beta\alpha_t^{bs} - \lambda_t^{bs}] = [0, \beta\alpha_t^{bs}]$. Likewise $\partial_y J^{SF}(z_t^s, z_t^s) = \beta[f_t - \alpha_t^s + H_t^{SF'}(z_t^s), f_t + \alpha_t^b + H_t^{SF'}(z_t^s)]$, and from (3.19) and (3.20) it follows that $-f_t - \alpha_t^b \leq H_t^{SF'}(z_t^s) \leq -f_t + \alpha_t^s$ for $y_t^b \leq z_t^s \leq y_t^s$, hence $\partial_y J_t^{SF}(z_t^s, z_t^s)$ ranges from $\beta[-\alpha_t^{bs}, 0]$ to $\beta[0, \alpha_t^{bs}]$ as z_t^s goes from y_s^b to y_t^s, hence $0 \in \partial_y J_t^{SF}(z_t^s, z_t^s)$.

Case (h-*iii*) $y_t^s < z_t^{cs}$**:** Now $y_t^* = y_t^s < z_t^* = z_t^{cs}$, and $\partial_y J_t^{SF}(z_t^{cs}, y_t^s) = \beta\left(f_t - \alpha_t^s + H_t^{SF'}(y_t^s)\right) = 0$ from (3.20). Likewise $\partial_z J_t^{SF}(z_t^{cs}, y_t^s) = K_t(z_t) + \beta\alpha_t^{bs} - \lambda_t^{bs}$, and z_t^{cs} as a solution to $K_t(z_t) = -\beta\alpha_t^{bs} + \lambda_t^{bs}$ as claimed in

(3.17). We can establish that $z_t^{cb} \le z_t^{cs} \le z_t^s$ by comparing (3.16), (3.17) and (3.18), and observing that K_t is a nondecreasing function.

Case (h-*iv*) $z_t^{cs} \le y_t^s \le z_t^s$**:** This theorem prescribes $z_t^* = y_t^* = y_t^s$, and from (3.20) it follows that $\partial_y J_t^{SF}(y_t^s, y_t^s) = \beta[f_t - \alpha_t^s + H_t^{SF'}(y_t^s), f_t + \alpha_t^b + H_t^{SF'}(y_t^s)] = \beta[0, \alpha^{bs}]$. The sub-differential $\partial_z J_t^{SF}(y_t^s, y_t^s) = \left[K_t(y_t^s) - \lambda_t^{bs}, K_t(y_t^s) - \lambda_t^{bs} + \beta\alpha_t^{bs}\right]$ It follows from cases (h-i) and (h-ii) that $\partial_z J_t^{SF}(y_t^s, y_t^s)$ goes from $[-\beta\alpha_t^{bs}, 0]$ for $y_t^s = z_t^{cs}$ to $[0, \beta\alpha_t^{bs}]$ for $y_t^s = z_t^s$.

Part (g): Consider the following four cases.

Case (g-*i*) $z_t^{cs} \le y_t^s < x_t$**:** The optimal policy prescribed by the theorem is $z_t^* = y_t^* = y_t^s$. The sub-differential $\partial_z J_t^{SF}(y_t^s, y_t^s) = \left[K_t(y_t^s) - \lambda_t^{bs}, K_t(y_t^s) - \lambda_t^{bs} + \beta\alpha_t^{bs}\right]$. It follows from (3.17) and (3.18) that this sub-differential ranges from $[-\beta\alpha_t^{bs}, 0]$ to $[0, \beta\alpha_t^{bs}]$ as x_t ranges from z_t^s to z_t^{cs}, hence $0 \in \partial_z J_t^{SF}(y_t^s, y_t^s)$. Similarly it follows from (3.20) that $0 \in \partial_y J_t^{SF}(y_t^s, y_t^s) = \beta[f_t - \alpha_t^s + H_t^{SF'}(y_t^s), f_t + \alpha_t^b + H_t^{SF'}(y_t^s)] = \beta[0, \alpha^{bs}]$.

Case (g-*ii*) $y_t^s < z_t^{cs} < x_t$**:** Now $z_t^* = z_t^{cs} > y_t^* = y_t^s$. It follows from (3.17) and (3.20) that $\partial_z J_t^{SF}(z_t^{cs}, y_t^s) = K_t(z_t^{cs}) - \lambda_t^{bs} + \beta\alpha_t^{bs} = 0$ and $\partial_y J_t^{SF}(z_t^{cs}, y_t^s) = \beta[f_t - \alpha_t^s + H_t^{SF'}(y_t^s)] = 0$.

Case (g-*iii*) $y_t^b \le x_t \le y_t^s$**:** In this case $z_t^* = y_t^* = x_t$ and $\partial_z J_t^{SF}(x_t, x_t) = \left[K_t(x_t) - \lambda_t^{bs}, K_t(x_t) + \beta\alpha_t^{bs}\right]$; this sub-differential ranges from $[-\beta\alpha_t^{bs}, \lambda_t^{bs}]$ to $[0, \lambda_t^{bs} + \beta\alpha_t^{bs}]$ as $z_t^* = x_t$ increases from z_t^{cs} to z_t^s, thus we conclude that $0 \in \partial_z J_t^{SF}(x_t, x_t)$. Similarly, $\partial_y J_t^{SF}(x_t, x_t) = \beta[f_t - \alpha_t^s + H_t^{SF'}(x_t), f_t + \alpha_t^b + H_t^{SF'}(x_t)]$ which ranges from $\beta[-\alpha_t^{bs}, 0]$ to $\beta[0, \alpha_t^{bs}]$ as $y_t^* = x_t$ ranges from y_t^b to y_t^s.

Case (g-*iv*) $x_t < y_t^b$**:** Now $z_t^* = x_t < y_t^* = y_t^b$, and

$\partial_z J_t^{SF}(x_t, y_t^b) = \left[K_t(x_t) - \lambda_t^{bs}, K_t(x_t)\right]$; this sub-differential ranges from $[-\beta\alpha_t^{bs}, \lambda_t^{bs} - \beta\alpha_t^{bs}]$ to $[0, \lambda_t^{bs}]$ as $z_t^* = x_t$ increases from z_t^{cs} to z_t^s. It follows from (3.19) that $\partial_y J_t^{SF}(x_t, y_t^b) = \beta[f_t + \alpha_t^b + H_t^{SF'}(y_t^b)] = 0$.

Part (f): Consider the following three cases.

Case (f-*i*) $y_t^s < x_t$**:** The prescribed policy is $y_t^* = y_t^s < z_t^* = x_t$, and the sub-differential is $\partial_z J_t^{SF}(x_t, y_t^s) = \left[K_t(z_t) - \lambda_t^{bs} + \beta\alpha_t^{bs}, K_t(z_t) + \beta\alpha_t^{bs}\right]$ which ranges from $[\beta\alpha_t^{bs} - \lambda_t^{bs}, \beta\alpha_t^{bs}]$ to $[0, \lambda_t^{bs}]$ as z_t increases from z_t^{cb} to z_t^{cs}, hence, $0 \in \partial J_t^{SF}(z_t)$. Similarly, it follows form (3.20) that $\partial_y J_t^{SF}(x_t, y_t^s) = \beta[f_t - \alpha_t^s + H_t^{SF'}(y_t^s)] = 0$.

Case (f-*ii*) $y_t^b \le x_t \le y_t^s$**:** Now the prescribed optimal policy is $z_t^* = y_t^* = x_t$, and the sub-differential $\partial_z J_t^{SF}(x_t, x_t) = \left[K_t(x_t) - \lambda_t^{bs}, K_t(x_t) + \beta\alpha_t^{bs}\right]$ which ranges from $[-\lambda_t^{bs}, \beta\alpha_t^{bs}]$ to $[-\beta\alpha_t^{bs}, \lambda_t^{bs}]$ as x_t increases from z_t^{cb} to z_t^{cs}, hence, $0 \in \partial_z J_t^{SF}(x_t, x_t)$. The sub-differential $\partial_y J_t^{SF}(x_t, x_t) = \beta[f_t - \alpha_t^s + H_t^{SF'}(x_t^s), f_t + \alpha_t^b + H_t^{SF'}(y_t^s)]$ which ranges from $[-\alpha_t^{bs}, 0]$ to $[0, \alpha_t^{bs}]$ as x_t increases from y_t^b to y_t^s.

Case (f-*iii*) $x_t < y_t^b$**:** Here the theorem prescribes $z^* = x_t < y_t^* = y_t^b$, and the sub-differential $\partial_z J_t^{SF}(x_t, y_t^b) = \left[K_t(x_t) - \lambda_t^{bs}, K_t(x_t)\right]$ which ranges from $[-\lambda_t^{bs}, 0]$ to $[-\beta\alpha_t^{bs}, \lambda_t^{bs} - \beta\alpha_t^{bs}]$ as x_t increases from z_t^{cb} to z_t^{cs}, hence, $0 \in \partial_z J_t^{SF}(x_t, y_t^b)$. It follows from (3.19) that $\partial_y J_t^{SF}(x_t, y_t^b) = \beta[f_t + \alpha_t^b + H_t^{SF'}(y_t^b)] = 0\diamondsuit$.

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